Aspects of Ising-nematic quantum critical point

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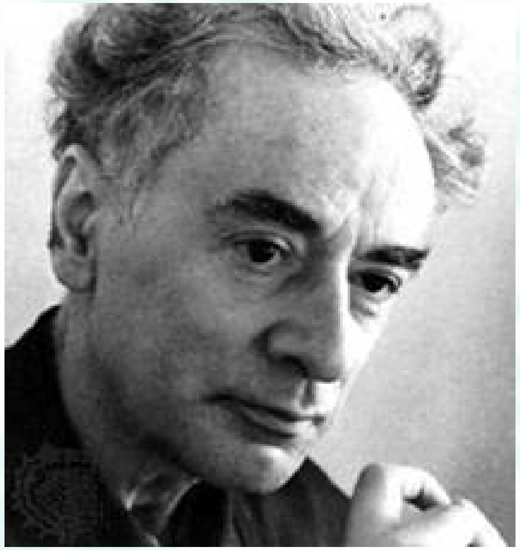
References

- UV/IR mixing in non-Fermi liquids, **Phys. Rev. B 92, 035141 (2015)**

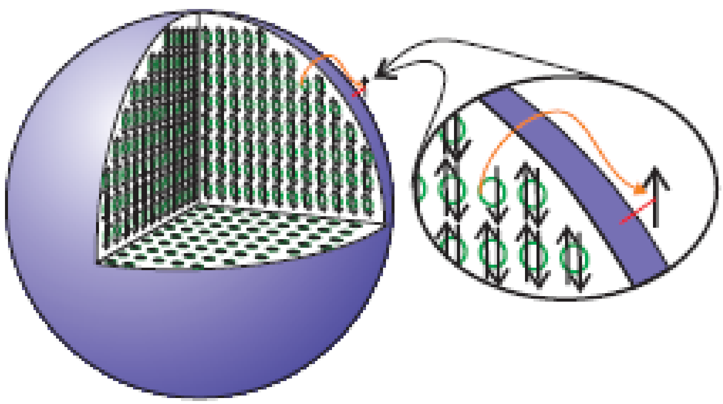
- Superconducting instability in non-Fermi liquids, **arXiv:1608.01320**

Landau Fermi-Liquid Theory



[Landau (1951)]: A finite density of interacting fermions doesn't depend on specific microscopic dynamics of individual systems :–

- **Ground state**: characterized by a sharp Fermi surface (FS) in momentum space
- **Low energy excitations**: weakly interacting quasi-particles around FS



$$(\omega = 0\,, \quad k_\perp \equiv k - k_F = 0)$$

$$\Downarrow$$

$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F\, k_\perp + i\,\Gamma}$$

❶ Quasiparticle lifetime diverges close to FS ☛ Decay rate $\Gamma \sim \omega^2$

❷ Electron has a finite overlap with quasiparticle adiabatically connected to non-interacting Fermi gas ☛ quasi-particle wt $Z > 0$

Breakdown of FL Theory

can be diagonalized in single-particle basis of quasiparticles

Fermi Liquid Metals

Low energy QFT

Non-Fermi Liquid States

no such basis

☛ genuinely interacting QFT

can arise when FS coupled with a gapless boson

gapless boson by fine-tuning microscopic parameters

Heavy fermion compounds near magnetic QCPs, QCP for Mott Transitions, nematic QCP

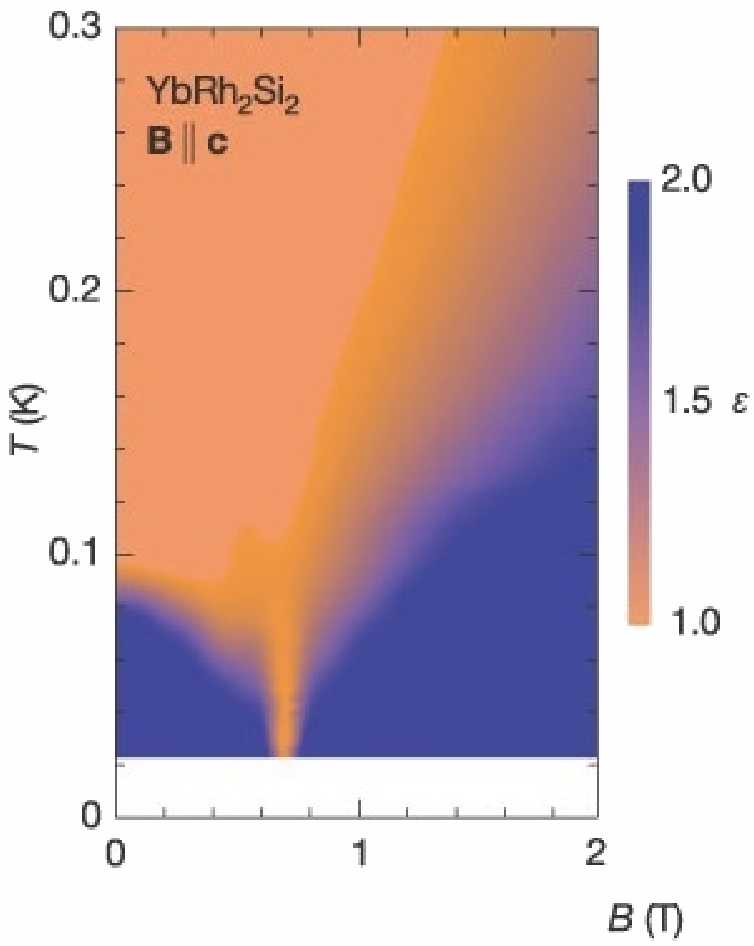
☛ NFL phase at QPT

gapless boson by dynamical tuning

Bose metals & ν = 1/2 FQH state support fractionalized fermionic excitations + emergent gauge field

☛ NFL phases in extended region in parameter space

Unusual Scaling Phenomenology



[Custers et al, Nature (2003)]

$[\rho(T) - \rho_0] \propto T^{\varepsilon}$

ε =1 for NFL (yellow)

ε =2 for FL (blue)

❶ Calculational framework that replaces FL theory needed.

❷ QFT of metals ☛ low symmetry + extensive gapless modes need to be kept in low energy theories ☛ less well understood compared to relativistic QFTs.

Goals

- Construct minimal field theories that capture universal low-energy physics.
- Understand the dynamics in controlled ways.
- Eventually come up with a systematic classification for NFL's.
- Broadly we have 2 cases:

critical boson mom $q = 0$
☛ Ising-nematic QCP,
gauge field + spinon FS

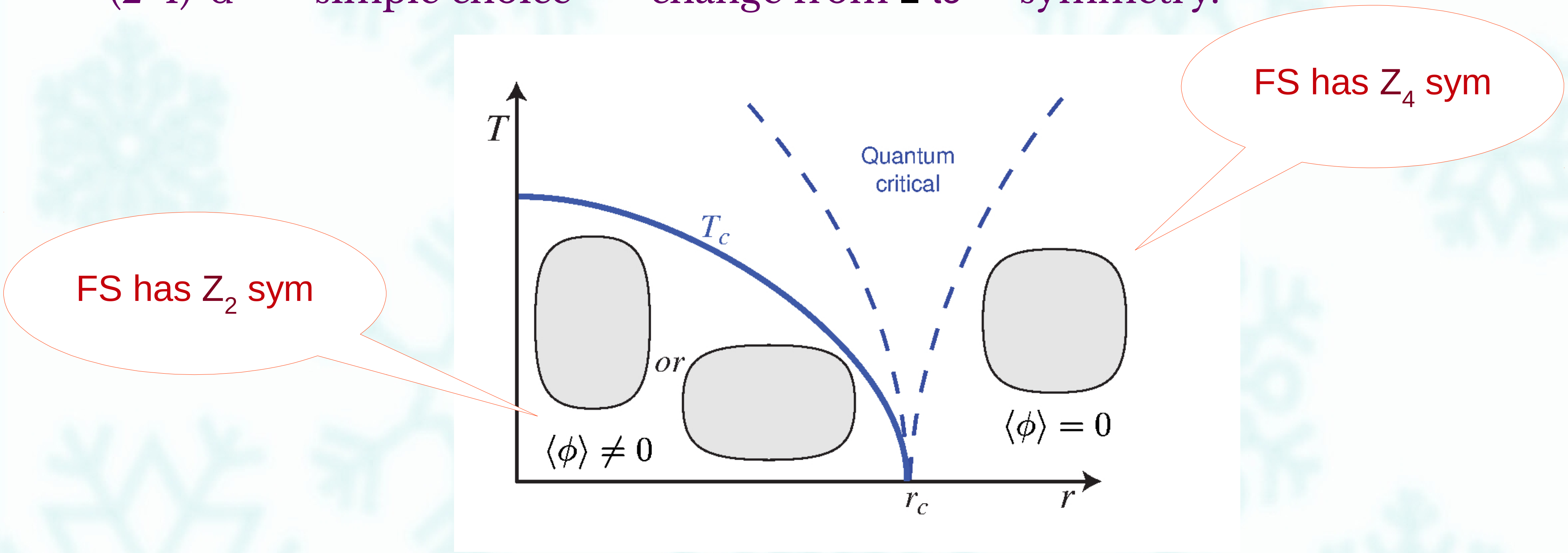
critical boson mom $q \neq 0$
☛ SDW or
CDW critical pts

- Dynamics depends on FS dim (m) in addition to spacetime dim (**d+1**). Here we focus on m & d-m dependence for case 1.

Ising-Nematic QPT

- From theoretical viewpoint ☛ Ising-nematic (ISN) QCP one of the simplest phase transitions in metals providing a remarkable strongly-coupled NFL with critical fluctuations of ISN order.

- (2+1)-d ☛ simple choice ☛ change from ■ to ▬ symmetry.



- QPT to nematic states with spontaneously broken point group symmetry ☛ order parameter is a real scalar boson with strong qtm fluctuations at QCP.

Dimension as a Tuning Parameter

- For d < upper critical dim d_c ☛ theory flows to interacting NFL at low energies.

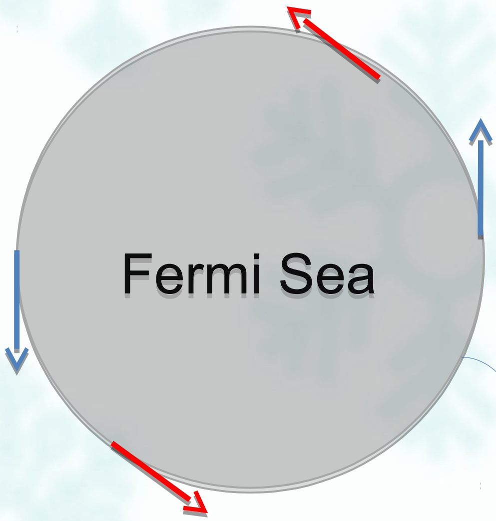
- For d > d_c ☛ expected to be described by FL .

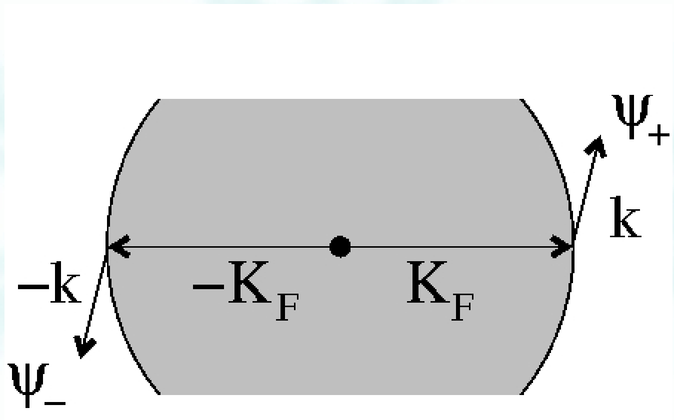
- Choice of regularization scheme for systematic RG in relativistic QFT :
  - ☛ Locality
  - ☛ Consistent with many symmetries

- Our Dimensional Regularization (DR) scheme:
  - ☛ Advantage ⇛ locality maintained

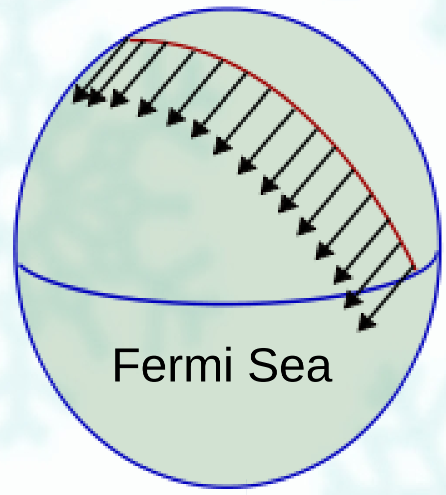
    [Locality broken in DR scheme of Senthil & Shankar (2009)]
  - ☛ Disadvantage ⇛ some symmetries broken [global U(1)]

Two Patch Theory





Time-Reversal Invariance assumed



Low energy limit

☛ Fermions coupled with boson with mom tangential to FS

☛ scatter tangentially

Not true for m-dim FS with m > 1

k_F enters as a dimensionful parameter

Circular FS (m=1) ☛ fermions in different patches decoupled except antipodal points

Significance of m for $d < d_c$

- d controls strength of qtm fluctuations & m controls extensiveness of gapless modes.

- For $d < d_c$ ☛ an emergent locality in mom space for $m = 1$, but not for $m > 1$.

- For $m = 1$ ☛ observables local in mom space (e.g. Green's fns) can be extracted from local patches ☛ need not refer to global properties of FS ☛ (2+1)-d ISN QCP described by a stable NFL state slightly below $d_c = 5/2$.

  [D. Dalidovich and S-S. Lee, Phys. Rev. B 88, 245106 (2013)]

- For $m > 1$ ☛ UV/IR mixing ☛ low-energy physics affected by gapless modes on entire FS ☛ effects patch theory cannot capture through renormalization of local properties.

Role of “ k_F ”

- We devise DR extending both **dim & co-dim** ☛ FS with m > 1 included naturally.

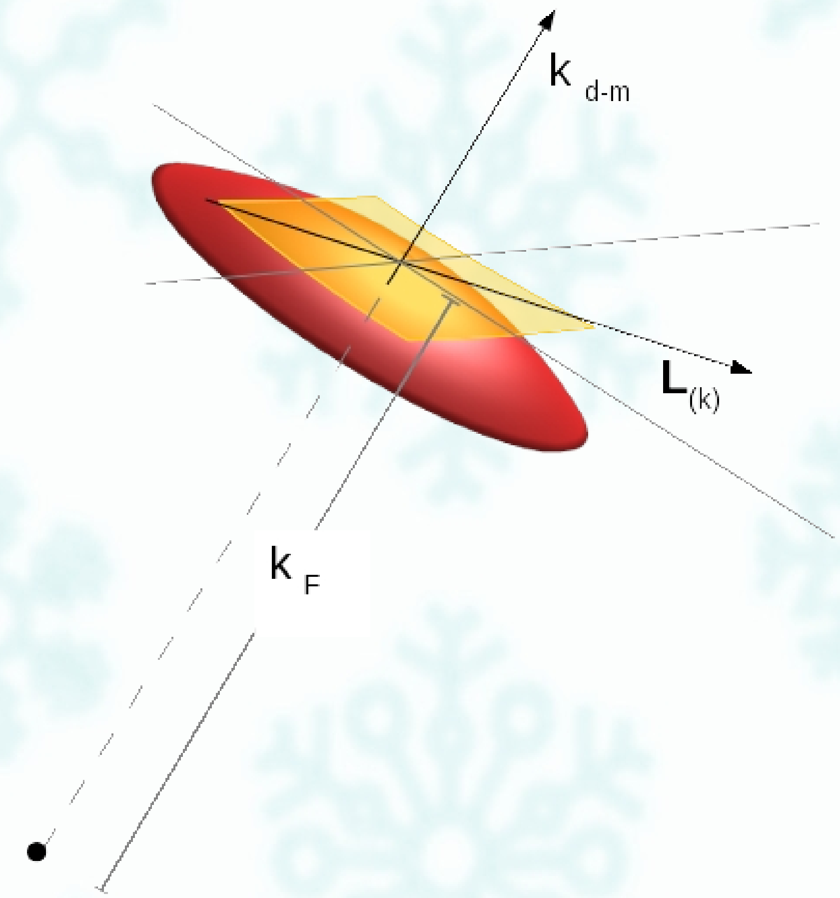
  [IM and S-S. Lee, PRB 92, 035141 (2015)]

- We provide a controlled analysis showing how interactions + UV/IR mixing interplay to determine low-energy scalings in NFL's with general m.

- For m > 1 ☛ size of FS (k_F) modifies naive scaling based on patch description ☛ k_F becomes a ‘naked scale’ .

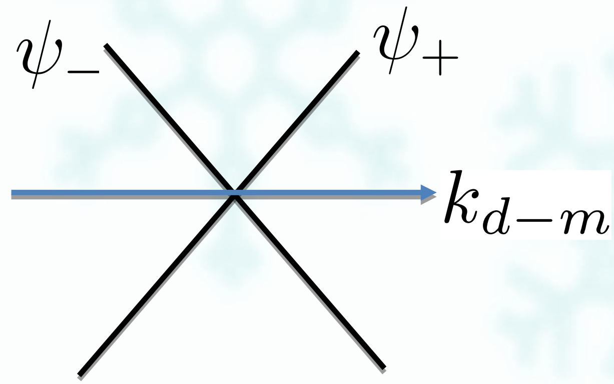
Generic Fermi Surface

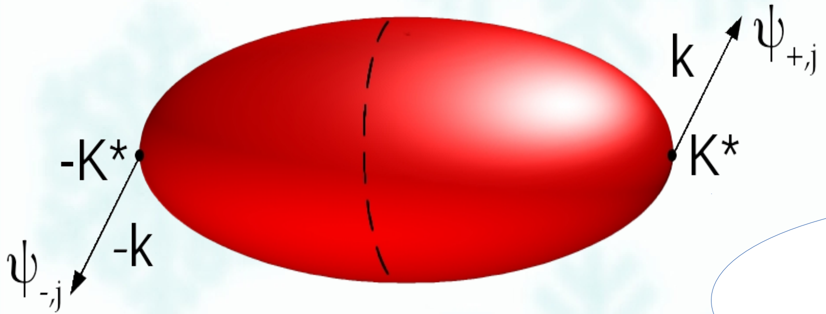
Patch of m-dim FS of arbitrary shape



- At a chosen point K^* on FS : $k_{d-m} \perp$ local S^m ☛ its magnitude measures deviation from k_F .
- $L_{(k)} = (k_{d-m+1} , k_{d-m+2} ,..., k_d)$ ☛ tangential along the local S^m.

Fermions on Antipodal Points





$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^{\dagger}_{-,j}(-k) \end{pmatrix}$$

$\psi_{+,j}\ (\psi_{-,j})$

right (left) moving fermion
with flavour j=1,2,...,N

Action

2 halves of m-dim FS
coupled with one critical boson
in (m+1)-space & one time dim:

$$
\begin{aligned}
S \quad = \quad & \sum_{s=\pm} \sum_{j=1}^{N} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \psi^{\dagger}_{s,j}(k) \left[i k_0 + s k_{d-m} + \vec{L}^2_{(k)} + \mathcal{O}(\vec{L}^3_{(k)}) \right] \psi_{s,j}(k) \\
+ \quad & \frac{1}{2} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \left[k_0^2 + k_{d-m}^2 + \vec{L}^2_{(k)} \right] \phi(-k)\, \phi(k) \\
+ \quad & \frac{e}{\sqrt{N}} \sum_{s=\pm} \sum_{j=1}^{N} \int \frac{d^{m+2}k\, d^{m+2}q}{(2\pi)^{2m+4}} \; \phi(q) \; \psi^{\dagger}_{s,j}(k+q)\, \psi_{s,j}(k)
\end{aligned}
$$

FS in Terms of Dirac Fermions

Interpret $|\mathbf{L}_{(k)}|$ as a continuous flavour

☛ **Each (m+2)-d spinor can be viewed as a (1+1)-d Dirac fermion**

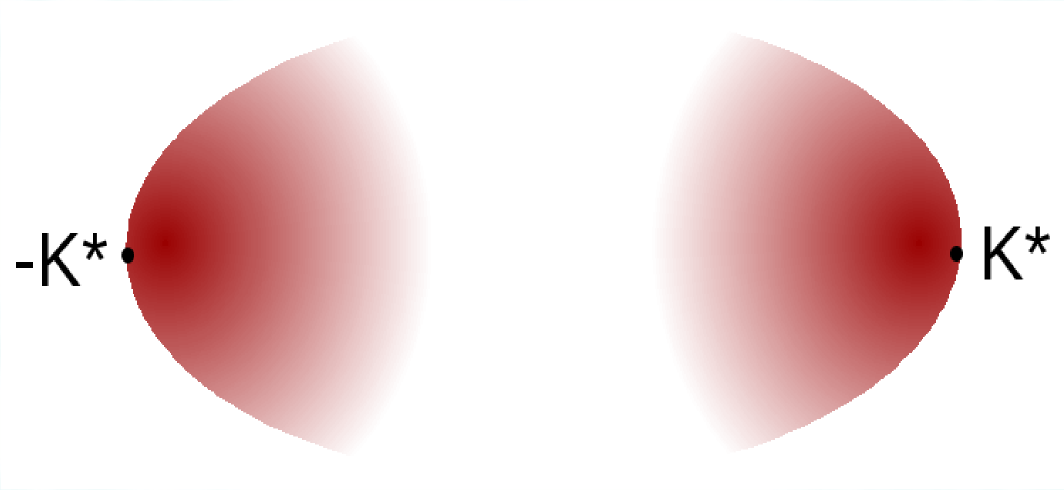
$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^\dagger_{-,j}(-k) \end{pmatrix}$$

$$S = \sum_{j=1}^{N} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \bar{\Psi}_j(k) \left[i k_0 \gamma_0 + i \left(k_{d-m} + \vec{L}^2_{(k)} \right) \gamma_1 \right] \Psi_j(k) \exp\left(\frac{\vec{L}^2_{(k)}}{k_F} \right)$$
$$+ \frac{1}{2} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \left[k_0^2 + k_{d-m}^2 + \vec{L}^2_{(k)} \right] \phi(-k)\, \phi(k)$$
$$+ \frac{ie}{\sqrt{N}} \sum_{j=1}^{N} \int \frac{d^{m+2}k\, d^{m+2}q}{(2\pi)^{2m+4}} \phi(q)\; \bar{\Psi}_j(k+q)\, \gamma_1\, \Psi_j(k)$$

mom cut-off

Momentum Regularization along FS

- Compact FS approx by 2 sheets of non-compact FS with a momentum regularization suppressing modes far away from $\pm K^*$:



- We keep dispersion parabolic but exp factor effectively makes FS size finite by damping $|\vec{L}_{(k)}| > k_F^{1/2}$ fermion modes.

Theory in General Dimensions

Add (d-m-1) spatial dim

☛ co-dimensions

$$\mathrm{k}_0 \to \vec{K} \equiv (k_0, k_1, \ldots, k_{d-m-1})$$

$$\gamma_0 \to \vec{\Gamma} \equiv (\gamma_0, \gamma_1, \ldots, \gamma_{d-m-1})$$

$$\gamma_1 \left(k_{d-m} + \vec{L}^2_{(k)}\right) \to \gamma_{d-m}\, \delta_k$$

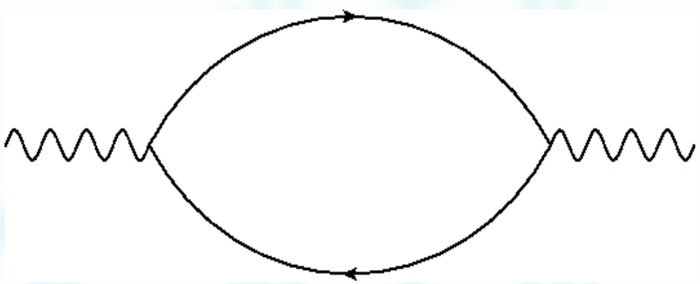
$$\delta_k = k_{d-m} + \vec{L}^2_{(k)}$$

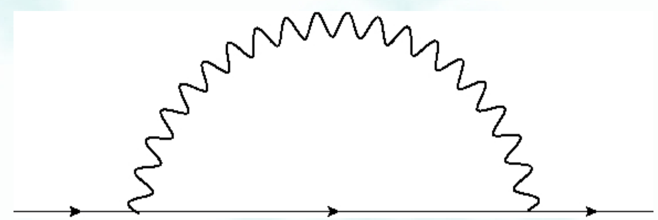
$$\begin{aligned} S \;=\; & \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \left[i\vec{\Gamma}\cdot\vec{K} + i\gamma_{d-m}\,\delta_k \right] \Psi_j(k) \\ + \; & \frac{1}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k_{d-m}^2 + \vec{L}^2_{(k)} \right] \phi(-k)\phi(k) \\ + \; & \frac{ie}{\sqrt{N}} \sum_j \int \frac{d^{d+1}k\, d^{d+1}q}{(2\pi)^{2d+2}} \phi(q) \bar{\Psi}_j(k+q) \gamma_{d-m} \Psi_j(k) \end{aligned}$$

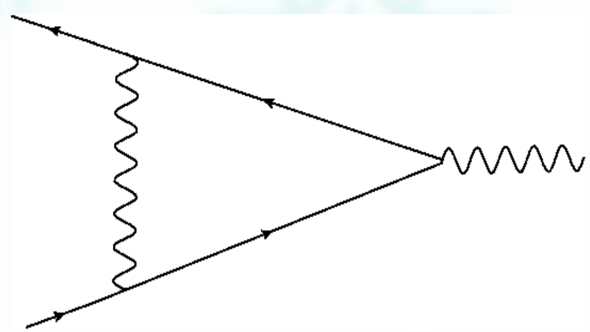
Applying DR

- There is an implicit UV cut-off Λ for K with $k << \Lambda << k_F$.
- k_F ☛ sets FS size;

  Λ ☛ sets the largest energy fermions can have $\perp$ FS .
- We consider RG flow by changing Λ & requiring low-energy observables independent of it.
- To access perturbative NFL, we fix **m** & tune **d** towards a critical dim d_c at which qtm corrections diverge logarithmically in Λ .







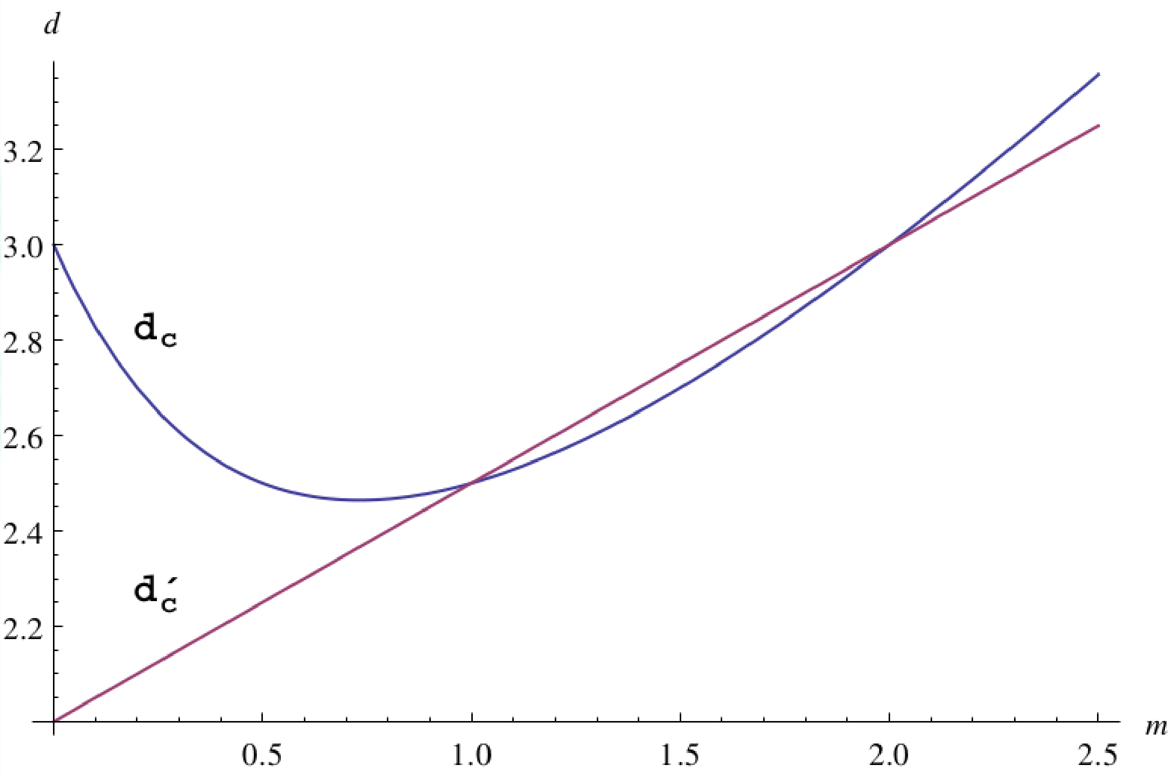
Critical Dimension

- Naïve critical dim ☛ scaling dim of e = 0 :

$$d'_c = \frac{4+m}{2}$$

- True critical dim ☛ one-loop fermion self-energy $\Sigma_1(q)$ blows up logarithmically :

$$d_c = m + \frac{3}{m+1}$$



One-Loop Results for d = $d_c - \epsilon$

Effective coupling
& control parameter in loop expansions

$$e_{eff} \equiv \frac{e^{2(m+1)/3}}{\tilde{k}_F^{\frac{(m-1)(2-m)}{6}}}$$

$$k_F = \mu\, \tilde{k}_F$$

Fixed points

$$\tilde{\beta} \equiv \frac{\partial e_{eff}}{\partial \ln \mu} = \frac{(m+1)(u_1\, e_{eff} - N\epsilon)\, e_{eff}}{3N - (m+1)u_1\, e_{eff}} = 0$$

Interacting Fixed Point

$$e^*_{eff} = \frac{N\epsilon}{u_1}$$

$$z^* = 1 + \frac{(m+1)\,\epsilon}{3}$$

$$\eta^*_\psi = \eta^*_\phi = -\frac{\epsilon}{2}$$

Dynamical critical exponent

Anomalous dimensions for fermions & boson

Stable NFL Fixed Point

Small e_{eff} expansion :

$$\tilde{\beta} = -\frac{(m+1)\epsilon}{3}\, e_{eff} + \frac{(m+1)\{3-(m+1)\epsilon\}\, u_1}{9N}\, e_{eff}^2 + \mathcal{O}(e_{eff}^3)$$

Low energy limit

☛ theory flows to

a

Stable

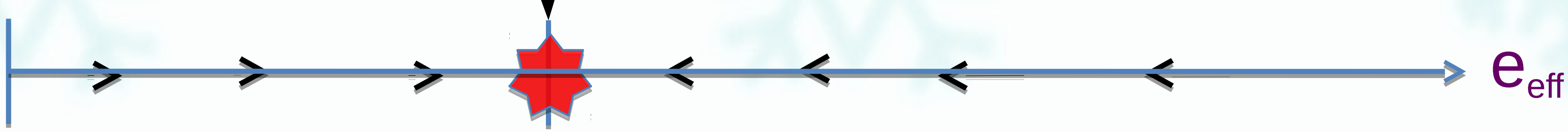
NFL

Fixed Point

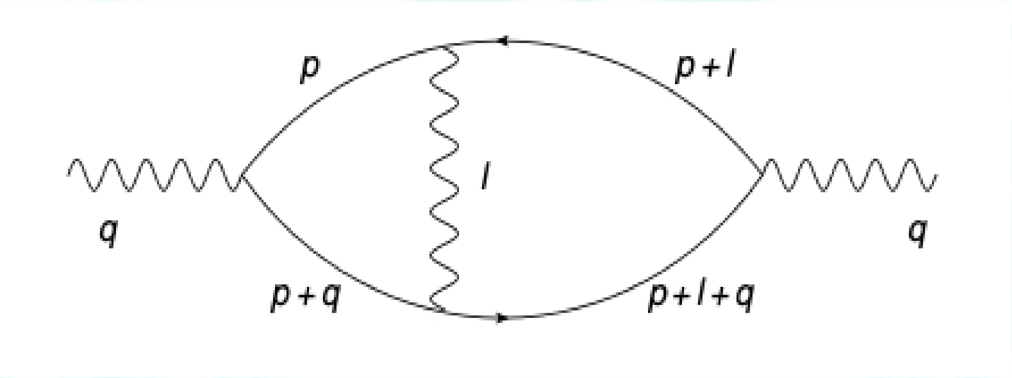
e_{eff} marginal at d_c

For small ϵ, interacting f.p. **perturbatively** accessible though e has +ve scaling dim for 1<m<2

RG Flow



Two-Loop Results : Boson Self-Energy



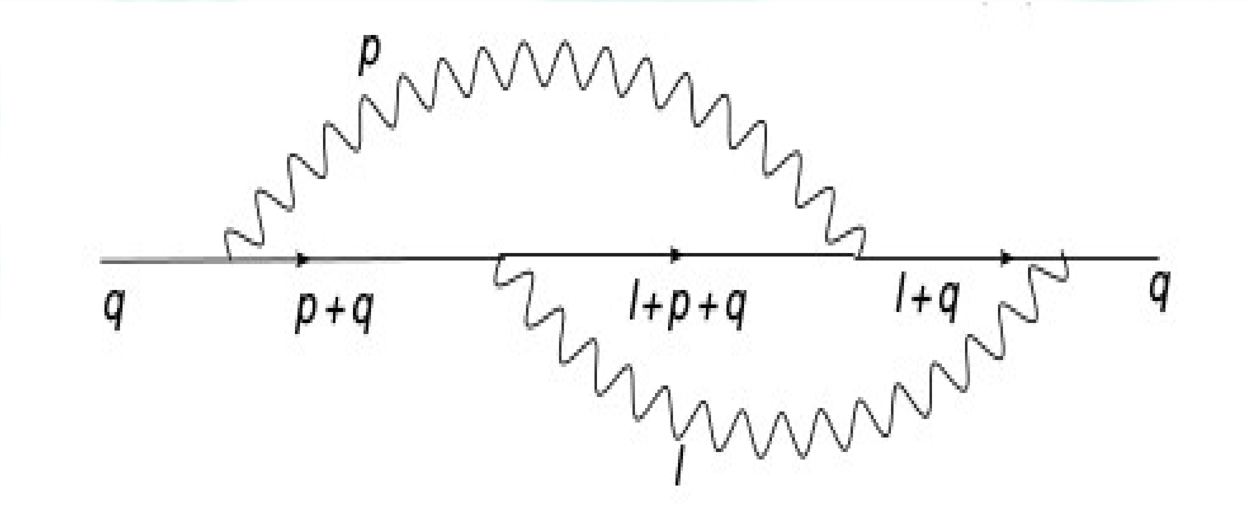
- For m > 1 ☛

$$\Pi_2(q) \quad \sim \quad \frac{e^2\, k_F^{\frac{m-1}{2}}\;\pi^2}{6\,N\,|\vec{L}_{(q)}|^2\;\sin\left(\frac{m\,\pi}{3}\right)}\,\frac{e_{eff}^{\frac{m}{m+1}}}{k_F^{\frac{m-1}{2\,(m+1)}}}$$

☛ k_F suppressed ☛ no correction at 2-loop

- For m = 1 ☛ UV-finite, gives a finite correction ☛ $\Pi_2(q) \sim \left(\frac{e^2}{N\,|L_{(q)}|}\right) e_{eff}$

Two-Loop Results : Fermion Self-Energy



- For m > 1 ☛ $\Sigma_2(q) \quad \sim \quad k_F - suppressed$

  ☛ no correction at 2-loop

- For m = 1 ☛ UV-divergent

Pairing Instabilities of Critical FS States

- Regular FL unstable to arbitrary weak interaction in BCS channel leading to Cooper pairing ☛ How about a critical FS ?

- Metlitski, Mross, Sachdev & Senthil [PRB 91, 115111 (2015)] ☛ studied SC instability in (2+1)-d for NFL ☛ perturbative control involved breaking locality

- Chung, IM, Raghu & Chakravarty [Phys. Rev. B 88, 045127 (2013)] ☛ found Hatree-Fock soln of self-consistent gap eqn for a FS coupled to a transverse U(1) gauge field in (3+1)-d.

- We want to consider ISN scenario for $m \geq 1$ using dimensional regularization ☛ locality mantained

[IM, arXiv:1608.01320]

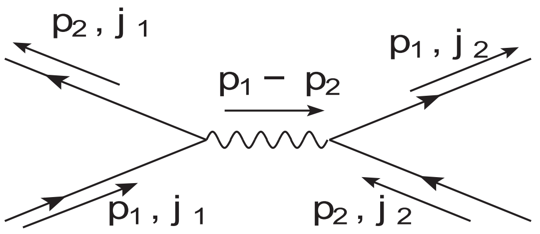
Superconducting Instability

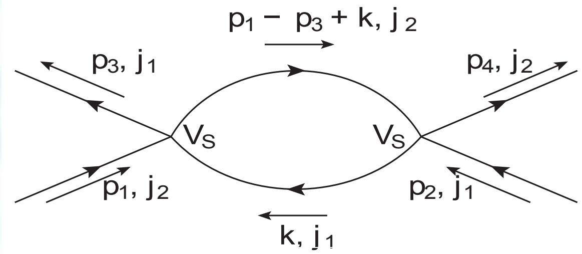
Add relevant 4-fermion terms
to analyse SC instability :

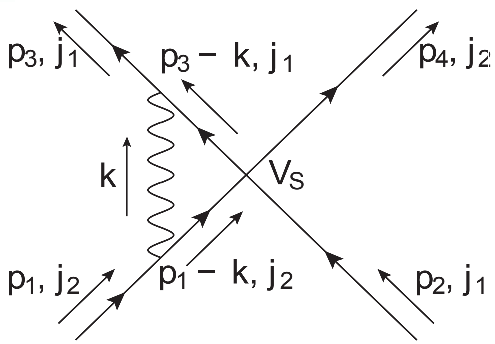
$$S^{\rm SC} = \frac{\mu^{d_v} V_S}{4} \sum_{j_1,j_2} \int \left(\prod_{s=1}^{4} dp_s \right) (2\pi)^{d+1}\, \delta^{(d+1)}(p_1+p_2-p_3-p_4)\, (\delta_{j_1,j_2}-1)$$
$$\times \Big[\{\bar\Psi_{j_1}(p_3)\Psi_{j_2}(p_1)\}\,\{\bar\Psi_{j_2}(p_4)\Psi_{j_1}(p_2)\} - \{\bar\Psi_{j_1}(p_3)\sigma_z\Psi_{j_2}(p_1)\}\,\{\bar\Psi_{j_2}(p_4)\sigma_z\Psi_{j_2}(p_1)\} \Big]$$

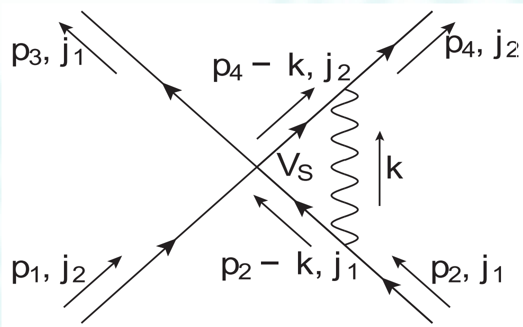
For simplicity , we consider **s-wave** case with **two** flavours

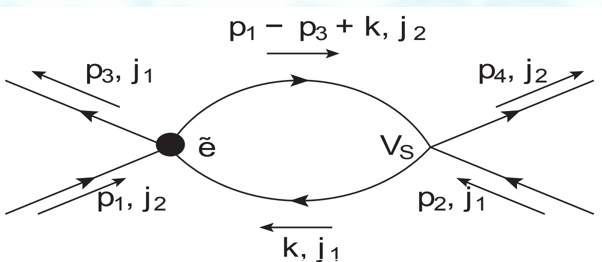
Feynman Diagrams

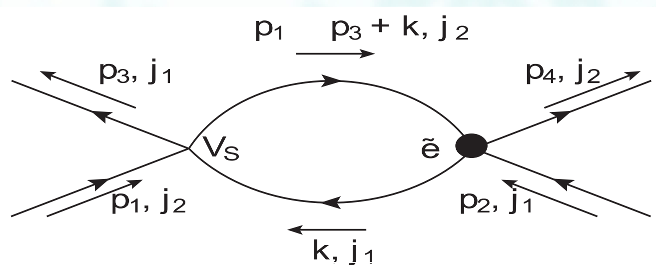












Beta-Fn for V_S

- Scatterings in pairing channel enhanced by volume of FS ~ $(k_F)^{m/2}$.

- Effective coupling that dictates potential instability :

$$\tilde{V}_S = \tilde{k}_F^{m/2} V_S$$

- $\tilde{V}_S$ marginal at co-dim d - m = 1.

- Aim ☛ study how e_{eff} affects pairing instability.

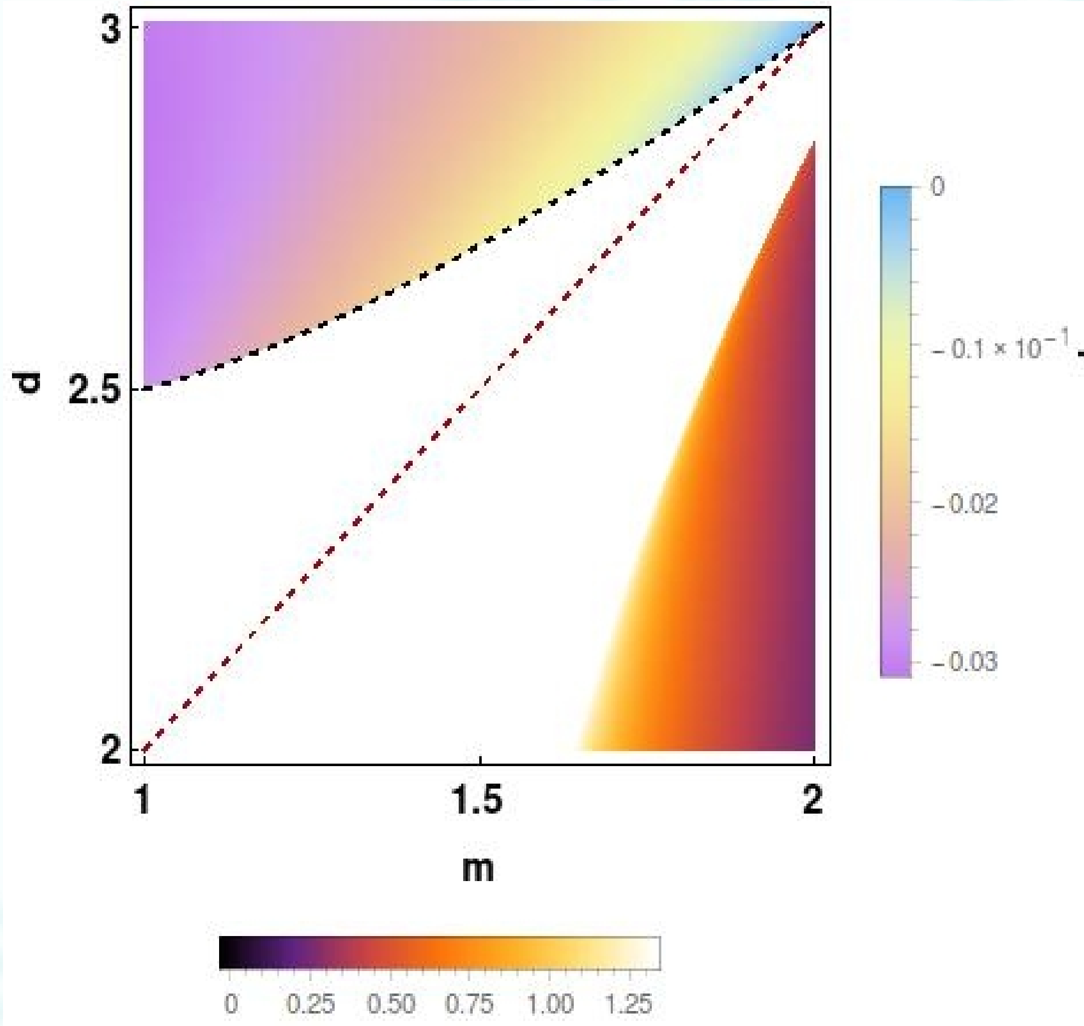
Beta-Fn for V_S ...

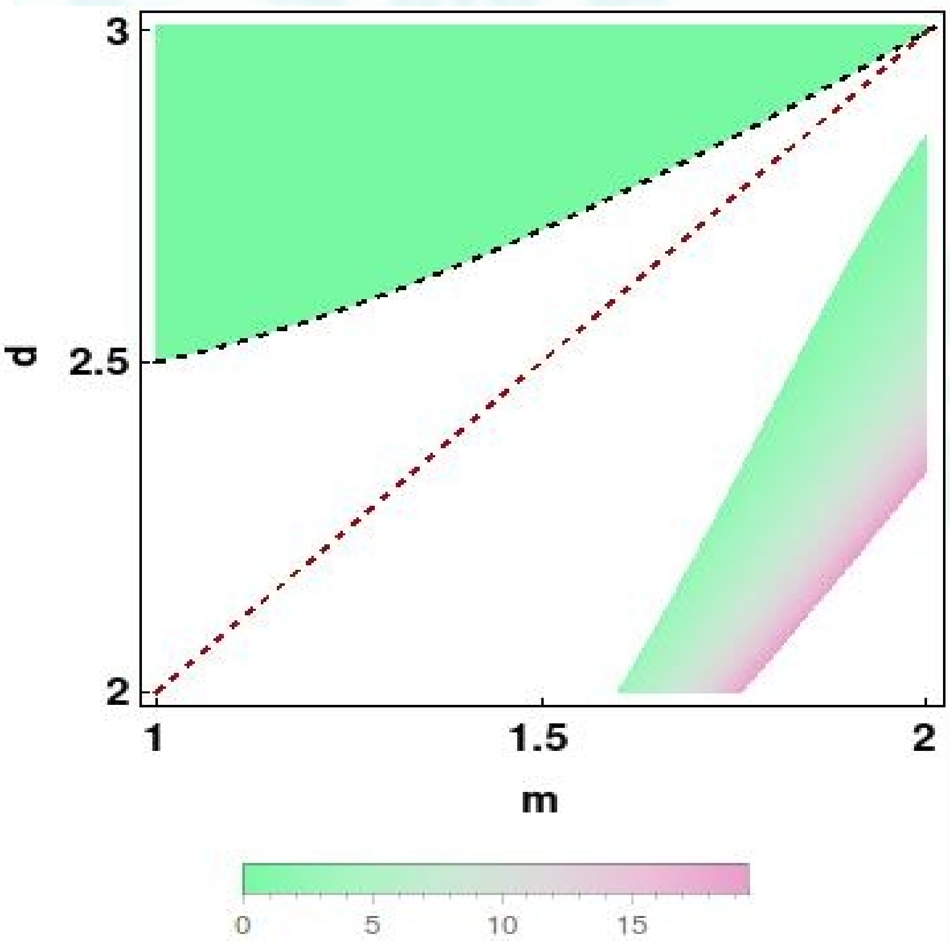
$$\frac{\partial \tilde{V}_S}{\partial l} = \gamma\,\epsilon\,\tilde{V}_S - v_2\,\tilde{V}_S^2 - v_1\,e_{eff} + v_3\,e_{eff}\,\tilde{V}_S$$

$$d - m = 1 - \gamma\,\epsilon$$

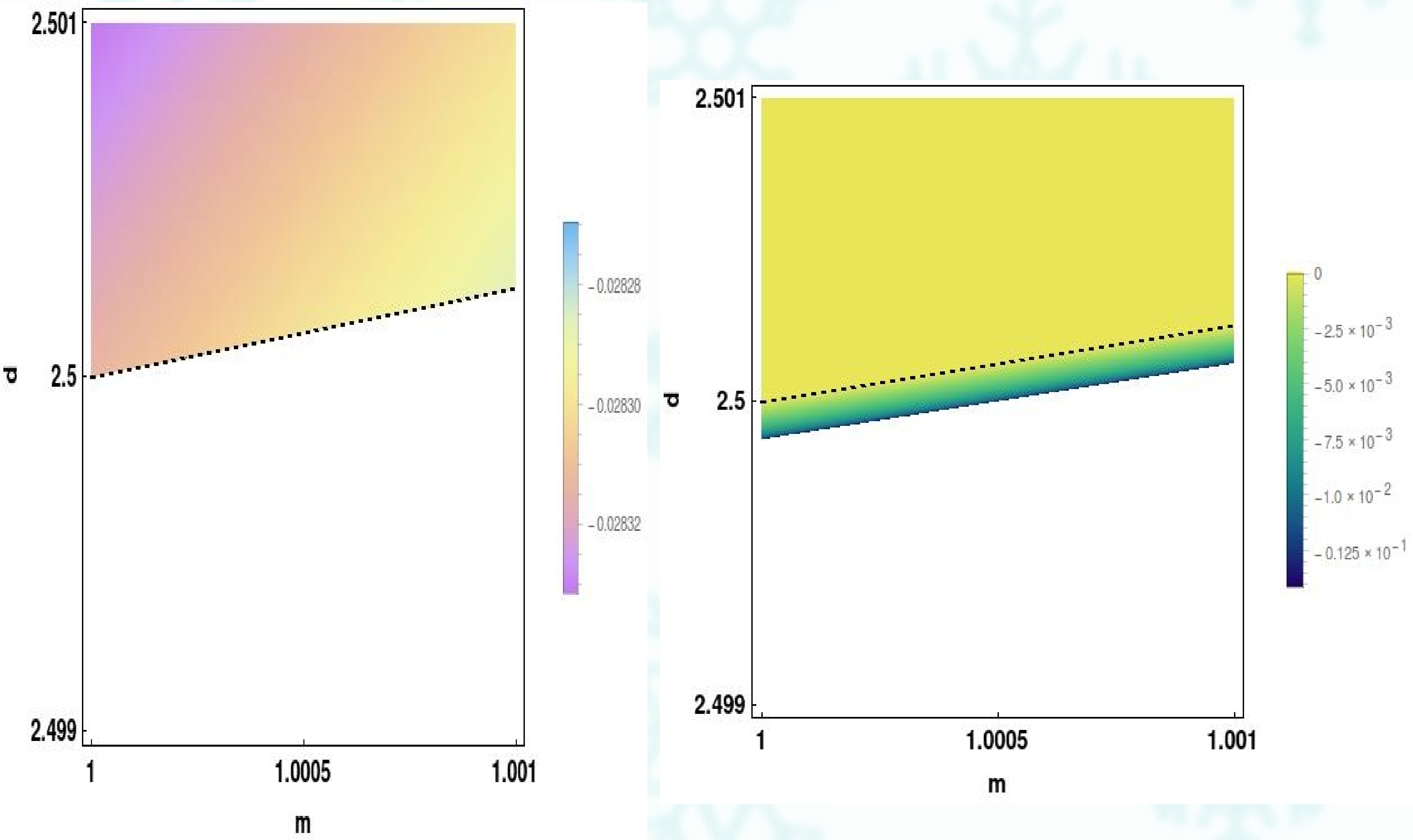
$$\gamma\,\epsilon = \epsilon - \frac{2-m}{m+1}$$

Solutions for $\tilde{V}_S$

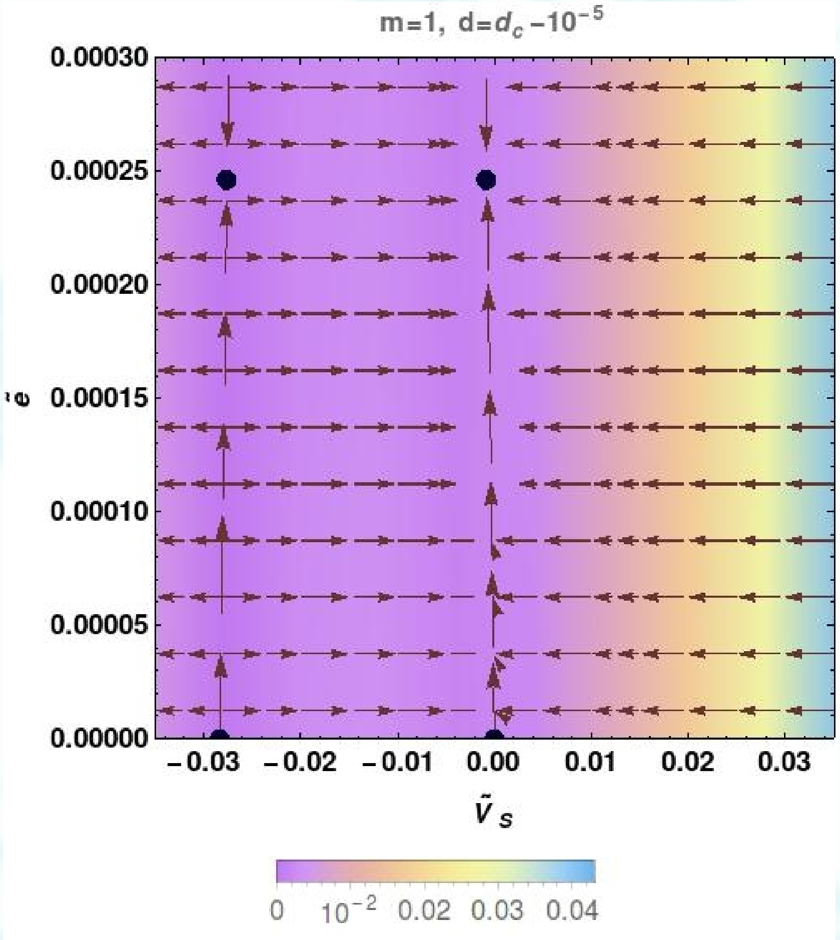


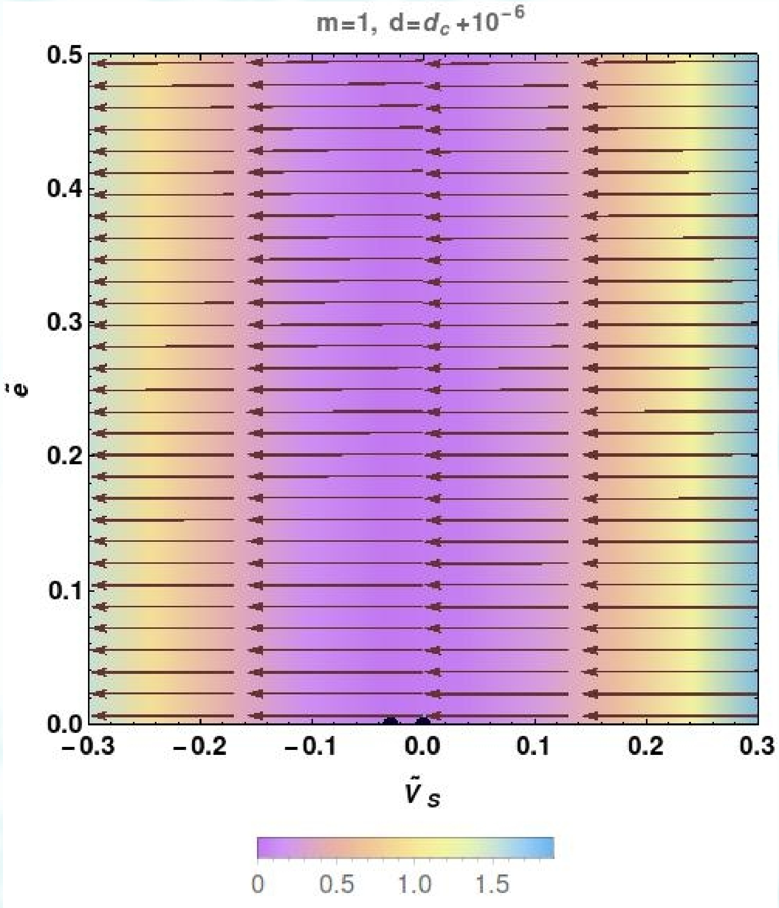


Solutions for $\tilde{V}_S$

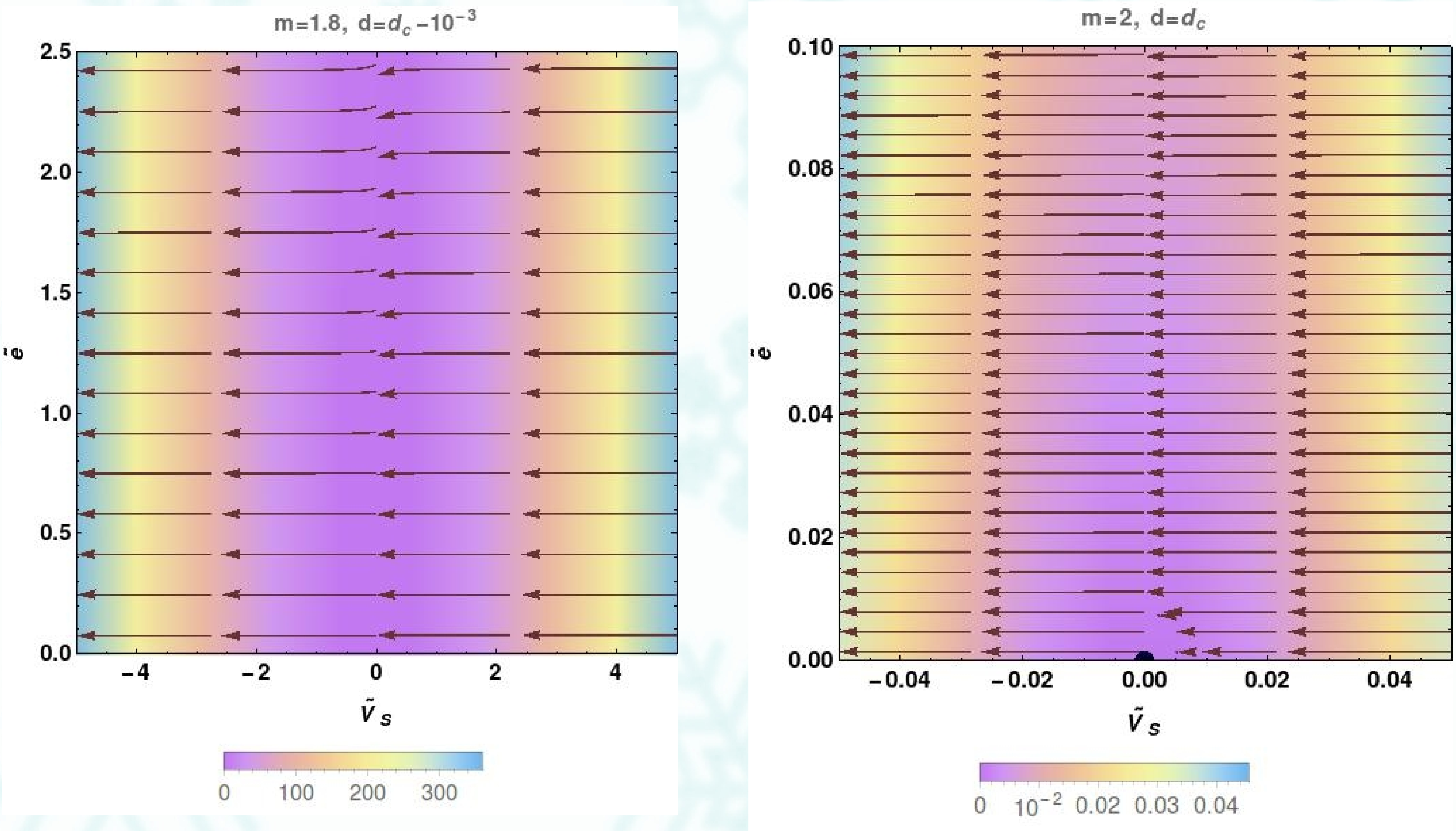


Fixed Points





Fixed Points



Epilogue

- RG analysis for QFTs with FS ☛ scaling behaviour of NFL states in a controlled approx.

- m-dim FS with its co-dim extended to a generic value ☛ stable NFL fixed points identified using $\epsilon = d_c - d$ as perturbative parameter.

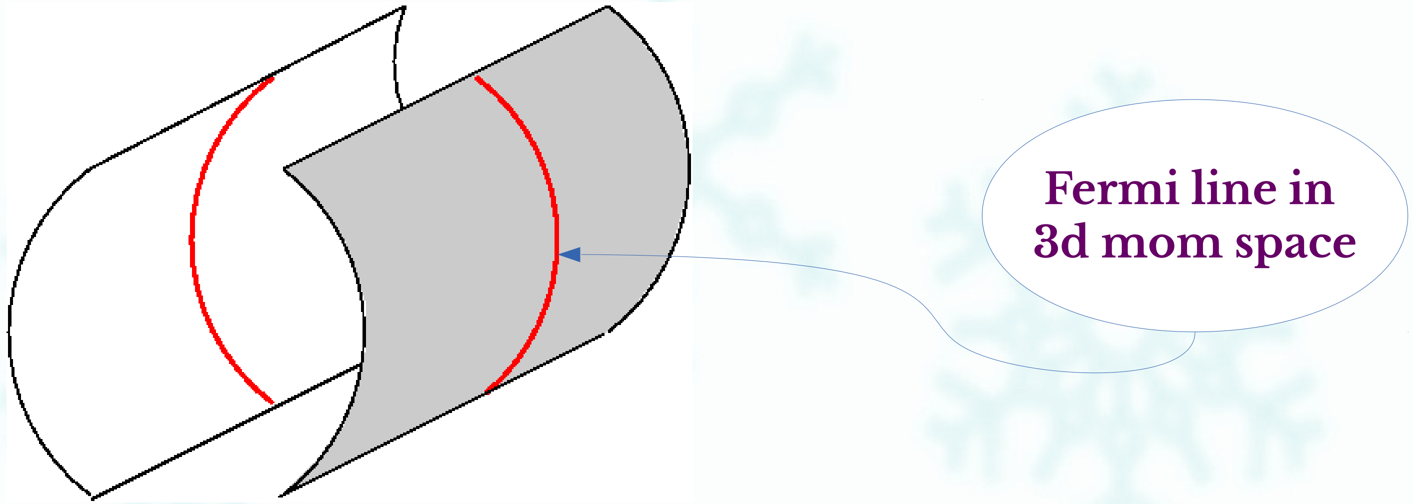
- SC instability in such systems as a fn of dim & co-dim of FS.

- Key point ☛ $\mathbf{k_F}$ enters as a dimensionful parameter unlike in relativistic QFT ☛ modify naive scaling arguments.

- Effective coupling constants ☛ combinations of original coupling constants & $\mathbf{k_F}$.

Thank you for your attention !

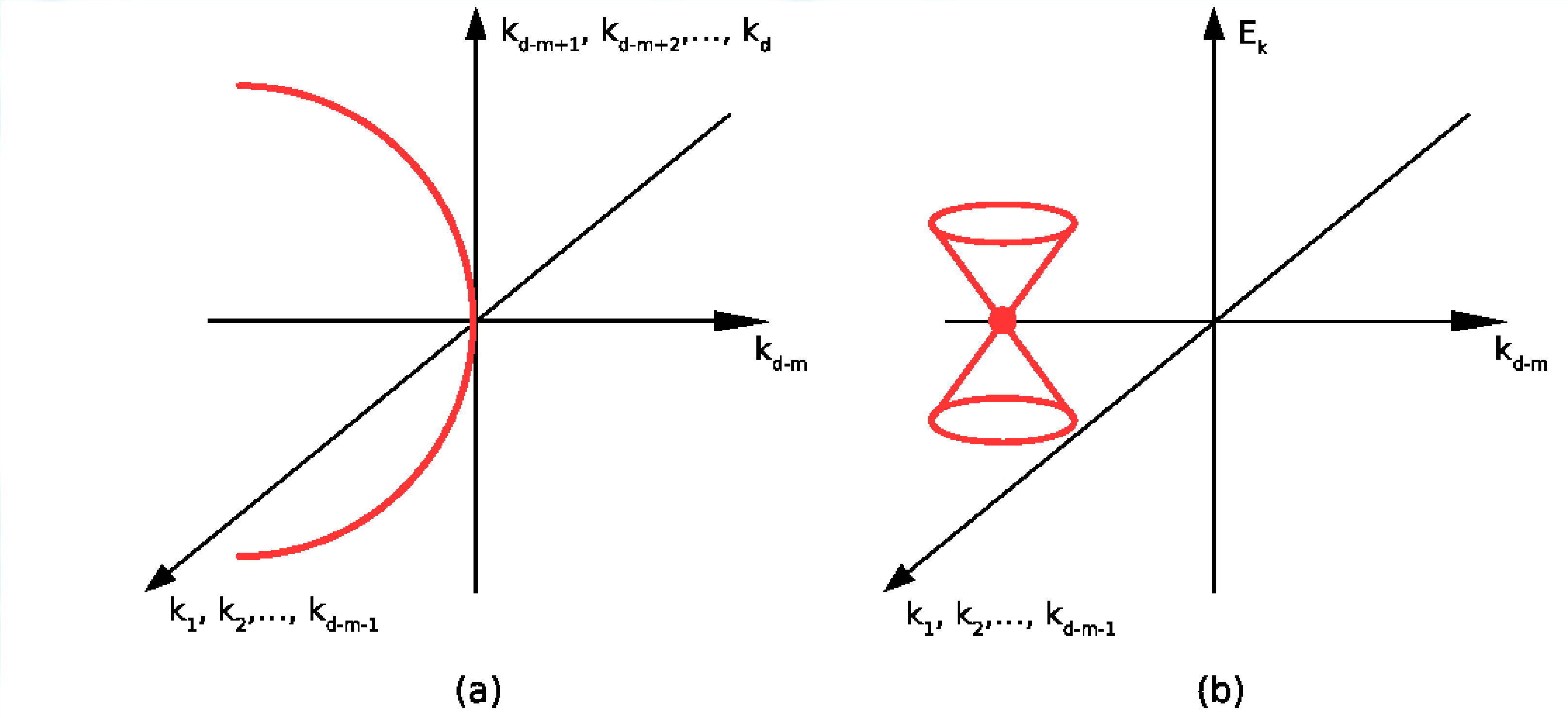
A Physical Realization for d=3, m=1



$$
\begin{aligned}
S &= \int \frac{d^4k}{(2\pi)^4} \Bigg\{ \sum_{s=\pm} \sum_{j=\uparrow,\downarrow} \psi^\dagger_{s,j}(k) \left(ik_0 + sk_2 + k_3^2 \right) \psi_{s,j}(k) \\
&\quad - k_1 \left(\psi^\dagger_{+,\uparrow}(k) \psi^\dagger_{-,\uparrow}(-k) + \psi^\dagger_{+,\downarrow}(k) \psi^\dagger_{-,\downarrow}(-k) + h.c. \right) \Bigg\}
\end{aligned}
$$

Turn on p-wave SC order parameter
☛ gap out the cylindrical FS
except for a line node

Line of Dirac Points



(a) m-dim FS embedded in d-dim mom space.

(b) Spinor has 2 bands: $$E_k = E_F \pm \sqrt{\sum_{i=1}^{(d-m-1)} k_i^2 + \delta_k^2}$$

For each $\mathbf{L}_{(k)}$ ☛ Dirac point ≡ $\left(k_1=0, k_2=0, \ldots, k_{d-m} = -(\mathbf{L}_{(k)})^2 \right)$ around which energy disperses linearly like a Dirac fermion in the (d-m)-dim subspace.

Two-point Fns at IR Fixed Point

- Using RG eqns ☛

$$\left\langle \phi(-k)\phi(k) \right\rangle = \frac{1}{\left(\vec{L}^2_{(k)}\right)^{2\Delta_\phi}} f_D\left(\frac{|\vec{K}|^{1/z^*}}{\vec{L}^2_{(k)}}, \frac{k_{d-m}}{k_F}, \frac{\vec{L}^2_{(k)}}{k_F} \right)$$

$$\left\langle \psi(k)\bar{\psi}(k) \right\rangle = \frac{1}{|\delta_k|^{2\Delta_\psi}} f_G\left(\frac{|\vec{K}|^{1/z^*}}{\delta_k}, \frac{\delta_k}{k_F}, \frac{\vec{L}^2_{(k)}}{k_F} \right)$$

- One-loop order ☛

$$f_D(x,y,z) = \left[1 + \beta_d\, \tilde{e}^{\frac{3}{m+1}}\, x^{\frac{3}{m+1}}\, z^{-\frac{3(m-1)}{2(m+1)}} \right]^{-1}$$

$$f_G(x,y,z) = -i \left[C\,(\vec{\Gamma}\cdot\hat{\vec{K}})\, x + \gamma_{d-m} \right]^{-1}$$