Bell's accelerating spaceships paradox and relativistic length contraction

Dedicated to the one hundredth anniversary of Hermann Minkowski's talk "Space and Time"

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Outline

- Introduction – 50 years have passed and the paradox still causes confusion. Why?

- Length contraction has nothing to do with the resolution of the paradox – the thread breaks due the increase of the proper distance between B and C, which is caused by the ***acceleration*** of B and C.

- A careful conceptual analysis makes the calculation of the increase of the proper distance BC very simple.

- The increase of the proper distance BC is not the ultimate explanation of why the thread breaks. The real explanation appears strange, but tells us something about the dimensionality of the objects involved in this paradox.

- Conclusion

“Three small spaceships, A, B, and C, drift freely in a region of space remote from other matter, without rotation and without relative motion, with B and C equidistant from A (Fig. 1).

On reception of a signal from A the motors of B and C are ignited and they accelerate gently...

Let ships B and C be identical, and have identical acceleration programmes. Then (as reckoned by an observer in A) they will have at every moment the same velocity, and so remain displaced one from the other by a fixed distance. Suppose that a fragile thread is tied initially between projections from B and C. If it is just long enough to span the required distance initially, then as the rockets speed up, it will become too short, because of its need to Fitzgerald contract, and must finally break. It must break when, at a sufficiently high velocity, the artificial prevention of the natural contraction imposes intolerable stress” (Bell 67).

John Stewart Bell
(28 June 1928 – 1 October 1990)
http://www.americanscientist.org/bookshelf/pub/john-bell-across-space-and-time

J. S. Bell: “How to teach special relativity”. In: *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge 1987) pp. 67-80

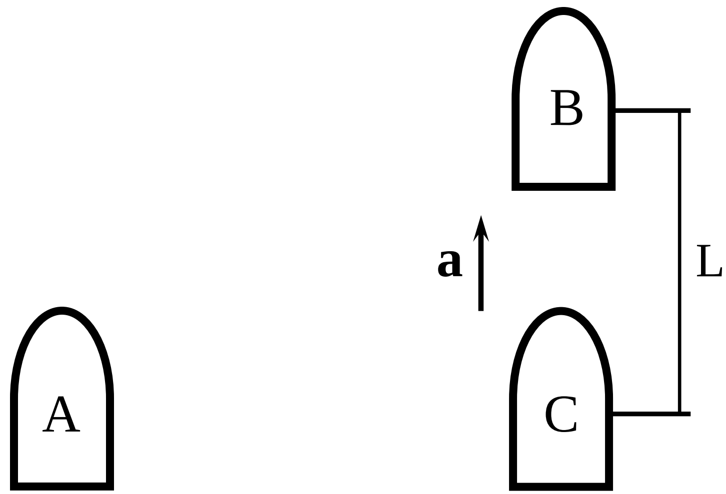

Figure 1

Three spaceships A, B, and C are initially at rest. At a given moment in A's reference frame B and C, which are connected with a thread, start to accelerate with the same proper acceleration. According to Bell, the thread will break due to stress caused by length contraction. His conclusion is based on the assumption that physical bodies contract relativistically, but space does not.

“This old problem came up for discussion once in the CERN canteen. A distinguished experimental physicist refused to accept that the thread would break, and regarded my assertion, that indeed it would, as a personal misinterpretation of special relativity. We decided to appeal to the CERN Theory Division for arbitration, and made a (not very systematic) canvas of opinion in it. There emerged a clear consensus that the thread would not break!” (Bell 68)

[3] Violent acceleration could break the thread just because of its own inertia while velocities are still small. This is not the effect of interest here. With gentle acceleration the breakage occurs when a certain *velocity* is reached, a function of the degree to which the thread permits stretching beyond its natural length (Bell 78).

E. Dewan, M. Beran: Note on **stress effects due to relativistic contraction**, Am. J. Phys. 27, 517-518 (1959)

A.A. Evett and R.K. Wangsness: Note on the **Separation** of Relativistically Moving Rockets, Am. J. Phys. 28, 521-606 (1960)

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E.M. Dewan, **Stress Effects due to Lorentz Contraction**, Am. J. Phys. 31 (5): 383-386 (1963)

A.A. Evett: A Relativistic Rocket Discussion Problem, Am. J. Phys. 40, 1170-1171 (1972)

"According to Lorentz any moving body must have undergone a contraction in the direction of its motion... This hypothesis sounds extremely fantastical, for the contraction is not to be looked upon as a consequence of resistances in the ether, or anything of that kind, but simply as a gift from above, - as an accompanying circumstance of the circumstance of motion" (Minkowski 81).

June 22, 1864 - January 12, 1909

http://de.wikisource.org/wiki/Raum_und_Zeit_(Minkowski)

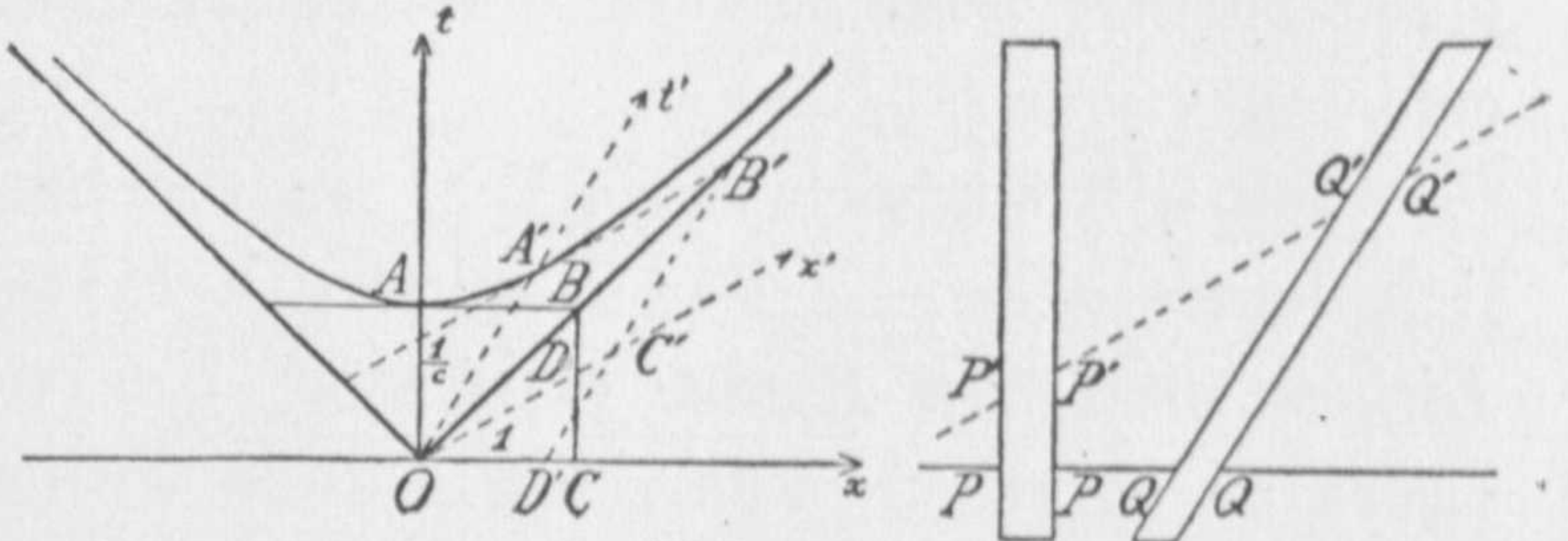


Fig. 1.

http://de.wikisource.org/wiki/Raum_und_Zeit_(Minkowski)

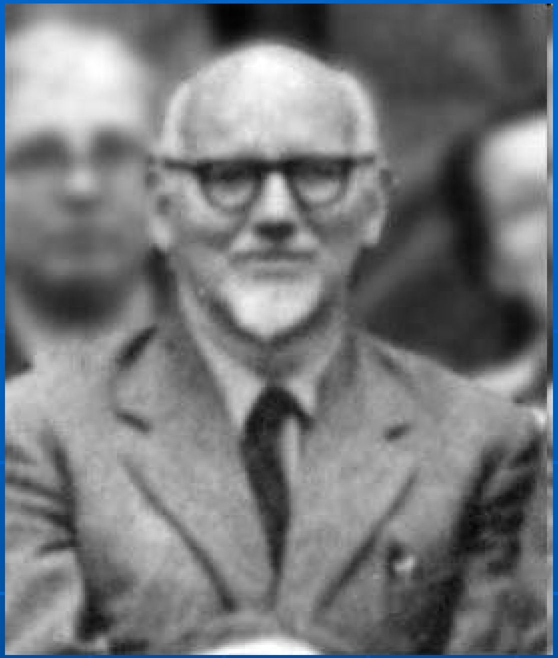
Unfortunately, many physicists do not take the reality of worldtubes of physical bodies seriously.

“The basic idea is to present the essentials of relativity from the Minkowskian point of view, that is, in terms of the geometry of space-time ... because it is to me (and I think to many others) the key which unlocks many mysteries. My ambition has been to make space-time a real workshop for physicists, and not a museum visited occasionally with a feeling of awe” (Synge 1965).

John Lighton Synge
23 March 1897 - 30 March 1995
http://www-groups.dcs.st-and.ac.uk/~history/PictDisplay/Synge.html

J.L. Synge: *Relativity: The Special Theory*, 2nd edn. (North-Holland, Amsterdam 1965) p. vii

"It is to support Minkowski's way of looking at relativity that I find myself pursuing the hard path of the missionary. When, in a relativistic discussion, I try to make things clearer by a space-time diagram, the other participants look at it with polite detachment and, after a pause of embarrassment as if some childish indecency had been exhibited, resume the debate in their own terms" (Synge 1960).



John Lighton Synge
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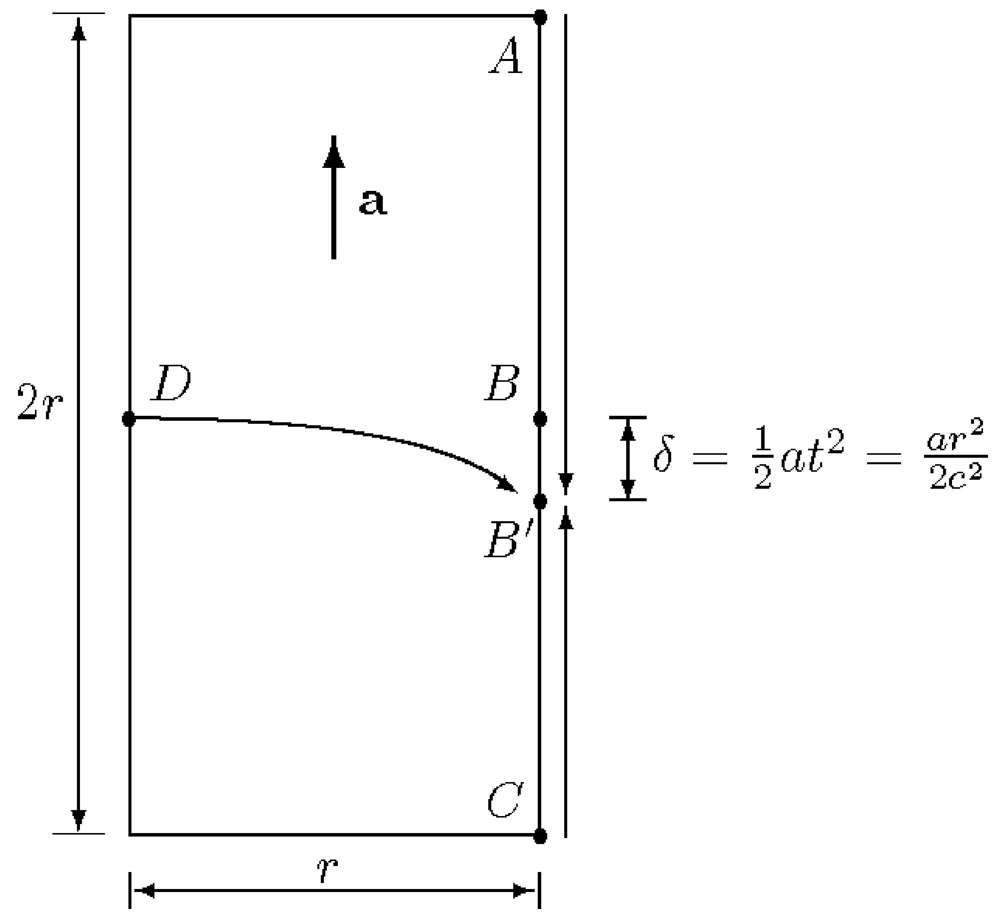
http://www-groups.dcs.st-and.ac.uk/~history/PictDisplay/Synge.html

J.L. Synge: *Relativity: The general theory* (North Holland Publishing Company, Amsterdam 1960), p. ix

Not taking the question of the reality of the absolute Minkowski four-dimensional world (or any relativistic spacetime) seriously causes confusion not only in the case of Bell’s paradox. It may also lead to missed opportunities for better understanding and perhaps even new results. Here are just two examples:

- Average velocity of light in non-inertial reference frames
    - Sagnac effect and Shapiro time delay
    - Anisotropic volume element – resolves two problems at once: (i) 1/2 factor in Fermi’s potential, and (ii) 4/3 factor in the self-force
- Origin of inertia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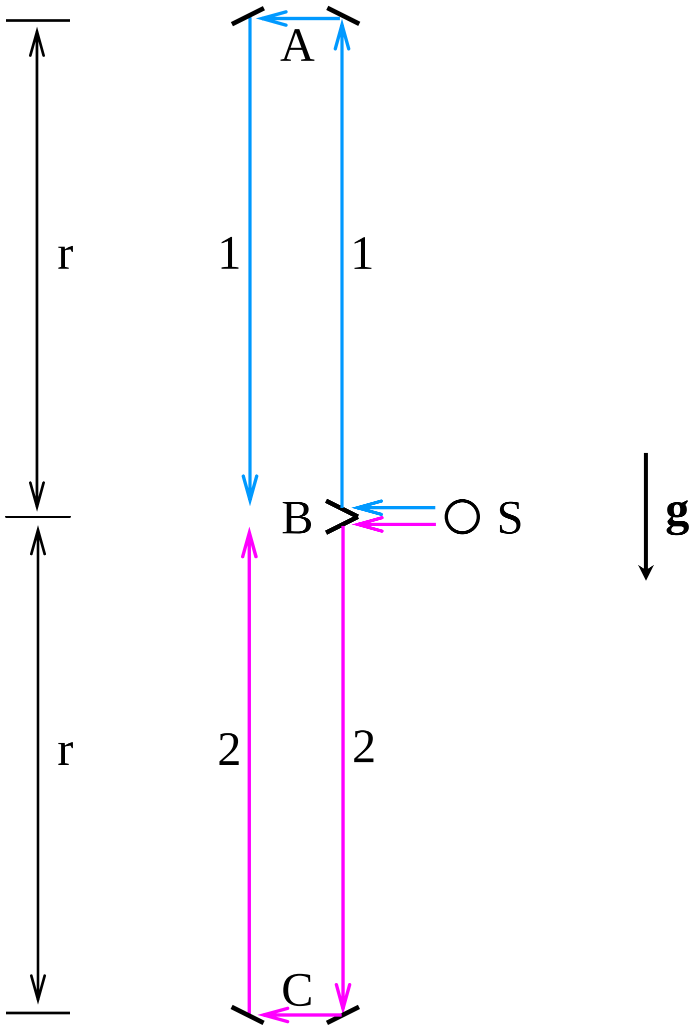

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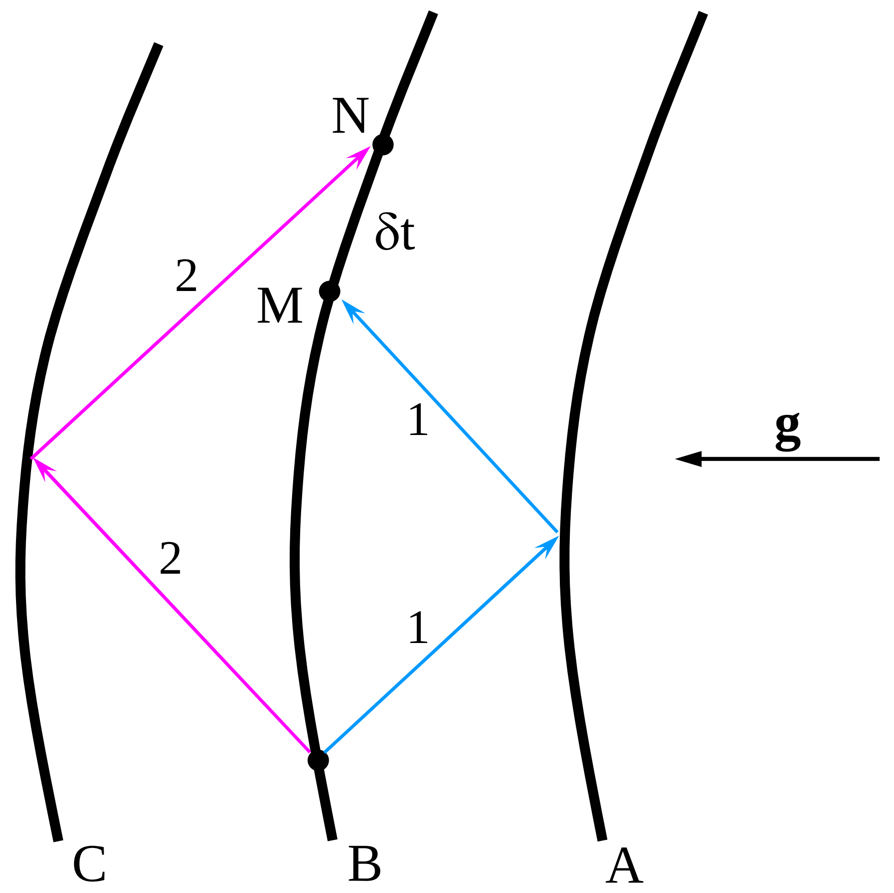

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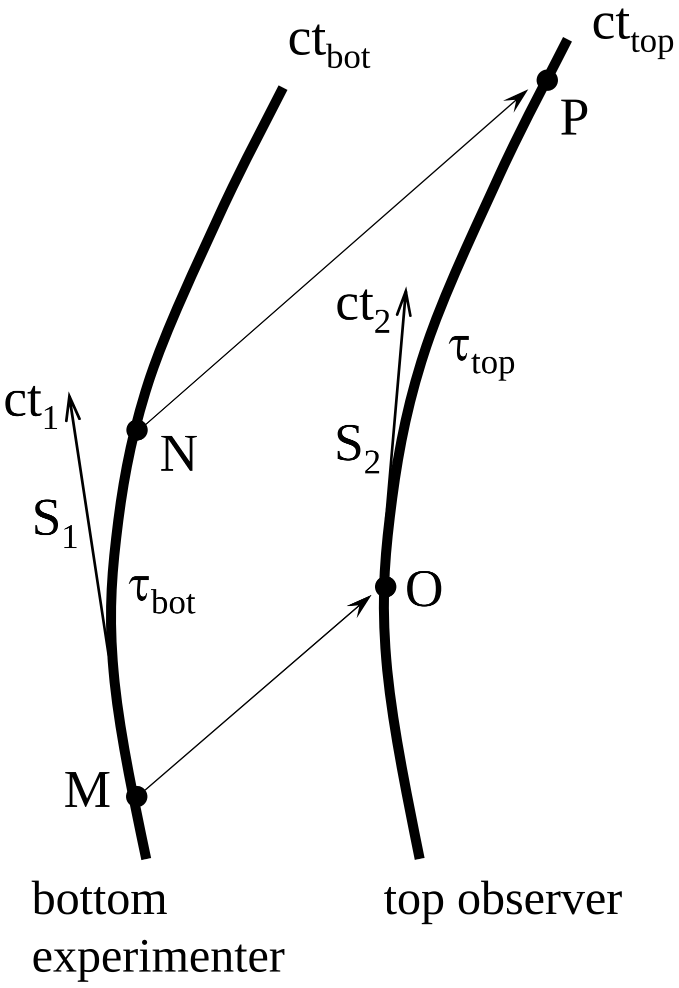
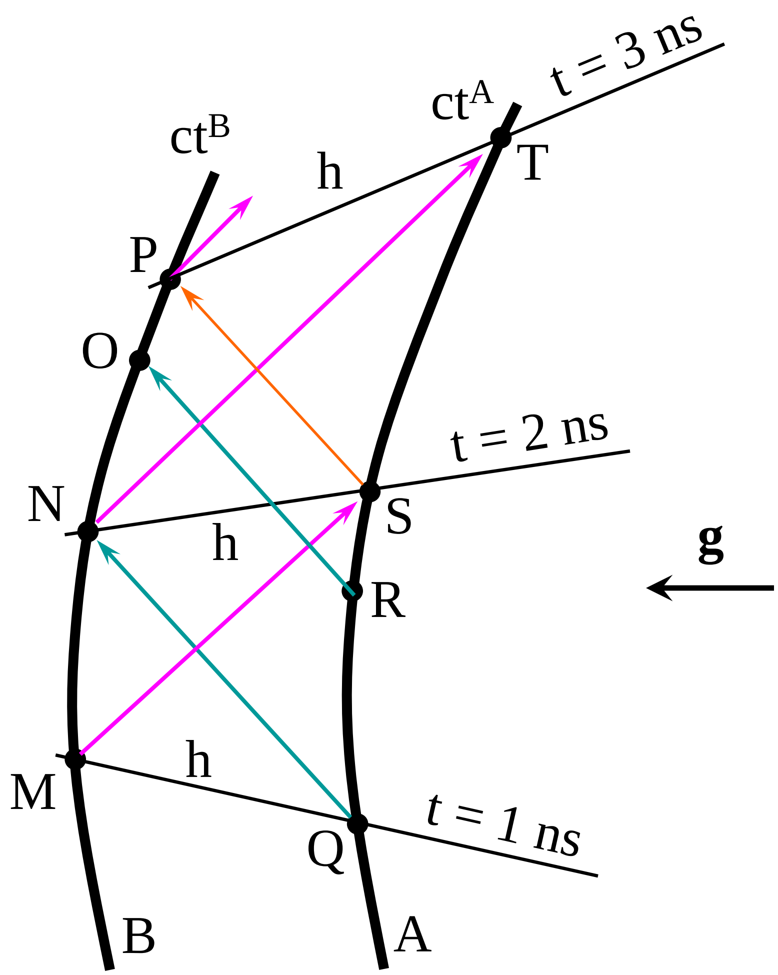
“One would again conclude, if flat Minkowski geometry were valid, that $\tau_{bot} = \tau_{top}$ thus contradicting the observed redshift experiment" (Misner et al. 189)



"Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's world-line (in most ordinary cases the latter predominate)" (Synge IX).



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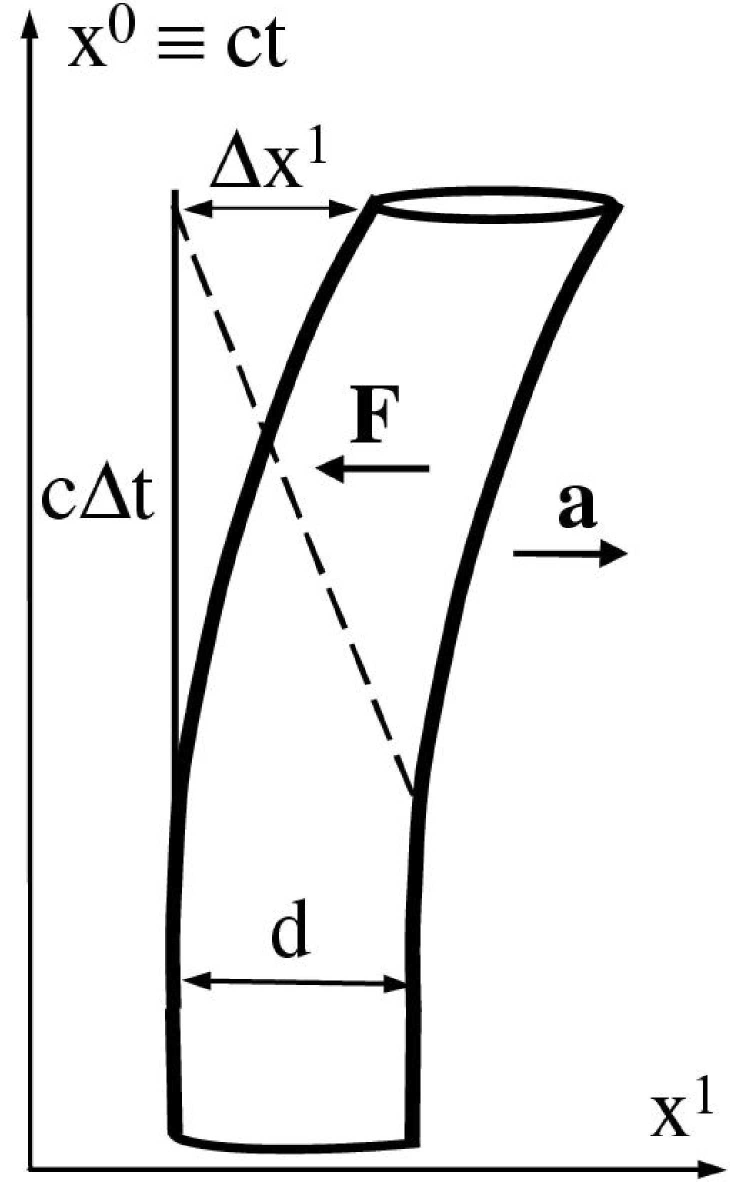


June 29, 2006

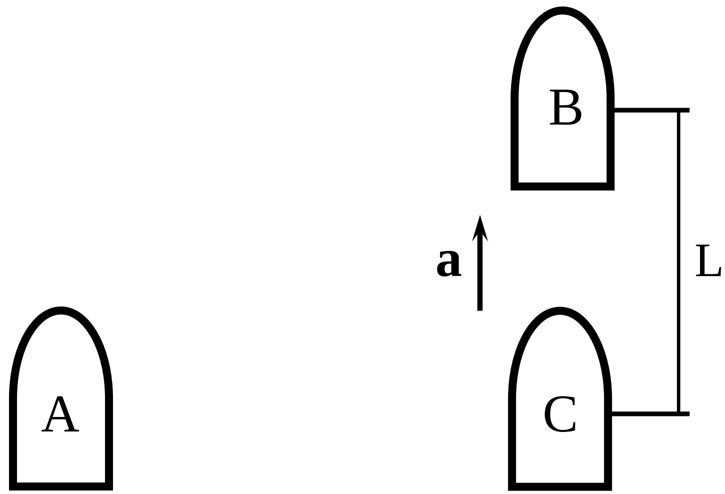


Figure 1

Three spaceships A, B, and C are initially at rest. At a given moment in A's reference frame B and C, which are connected with a thread, start to accelerate with the same proper acceleration. According to Bell, the thread will break due to stress caused by length contraction. His conclusion is based on the assumption that physical bodies contract relativistically, but space does not.

Bell's explanation is based on a pre-relativistic intuition and is incorrect on five counts:

- The thread contracts but space does not.

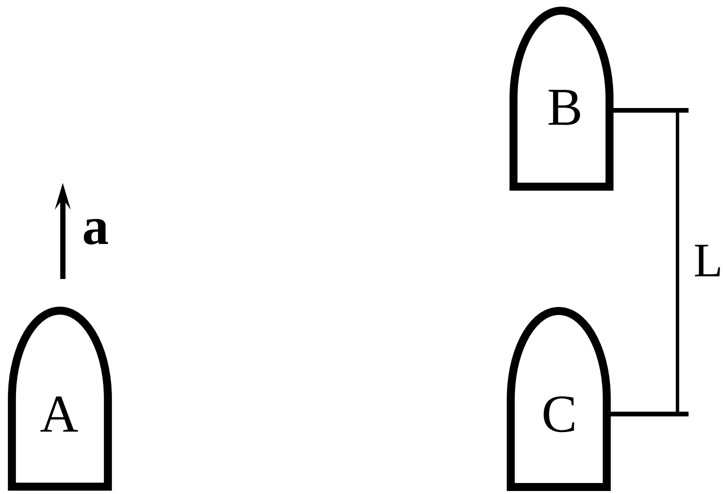
- The thread breaks as a result of a stress caused by relativistic length contraction.

- The thread breaks due to length contraction.

- If the reason for the break of the thread were length contraction, the proper length of the thread must be constant.

- All observers A, B and C measure the *same* three-dimensional thread.

3. The thread does not break due to length contraction.



Three spaceships A, B, and C are initially at rest. But unlike the thought experiment depicted in Fig. 1 it is spaceship A that starts to accelerate in this case. If the thread in Fig. 1 broke not because of the acceleration of B and C, but because of stress caused by length contraction, then in the case depicted here an observer in A would conclude that the thread should also break. However, that would be a total mystery for observers in B and C. So taking into account the ***relativity of motion*** in the length contraction effect demonstrates that no stress is involved in this effect and therefore the thread in Fig. 1 does not break because of length contraction.

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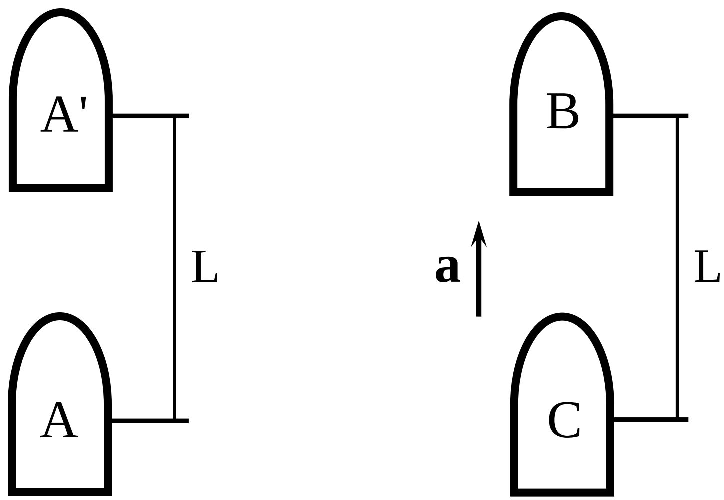
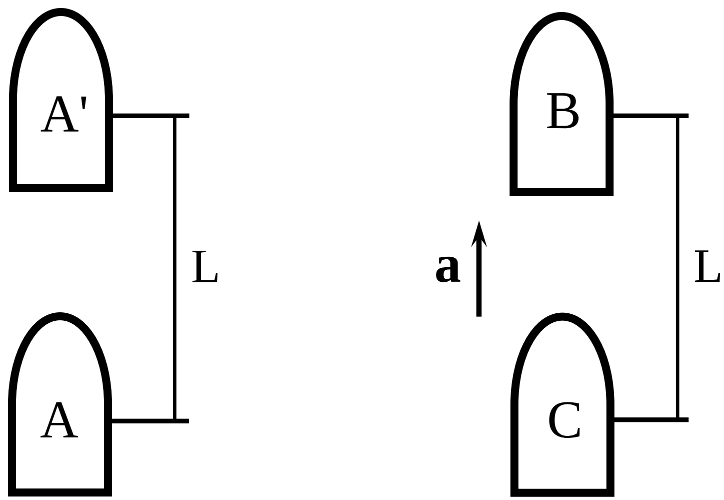


Another spaceship A' is added to the thought experiment depicted in Fig. 1. Spaceships A and A' are also connected with a thread. If the thread in Fig. 1 broke not because of the acceleration of B and C, but because of stress caused by length contraction, then observers in B and C would conclude that the thread connecting A and A' should also break. However, that would be a total mystery for observers in A and A'. So taking into account the ***reciprocity of length contraction*** demonstrates that no stress is involved in this effect and therefore the thread in Fig. 1 does not break because of length contraction.

4. If the reason for the break of the thread were length contraction, the proper length of the thread must be constant. Bell admits it is not:

“Of course many people who give this wrong answer at first get the right answer on further reflection. Usually they feel obliged to work out how things look to observers B or C. They find that B, for example, sees C drifting further and further behind, so that a given piece of thread can no longer span the distance. It is only after working this out, and perhaps only with a residual feeling of unease, that such people finally accept a conclusion which is perfectly trivial in terms of A's account of things, including the Fitzgerald contraction” (Bell 68).

Length contraction implies that the proper distance remains ***constant***.



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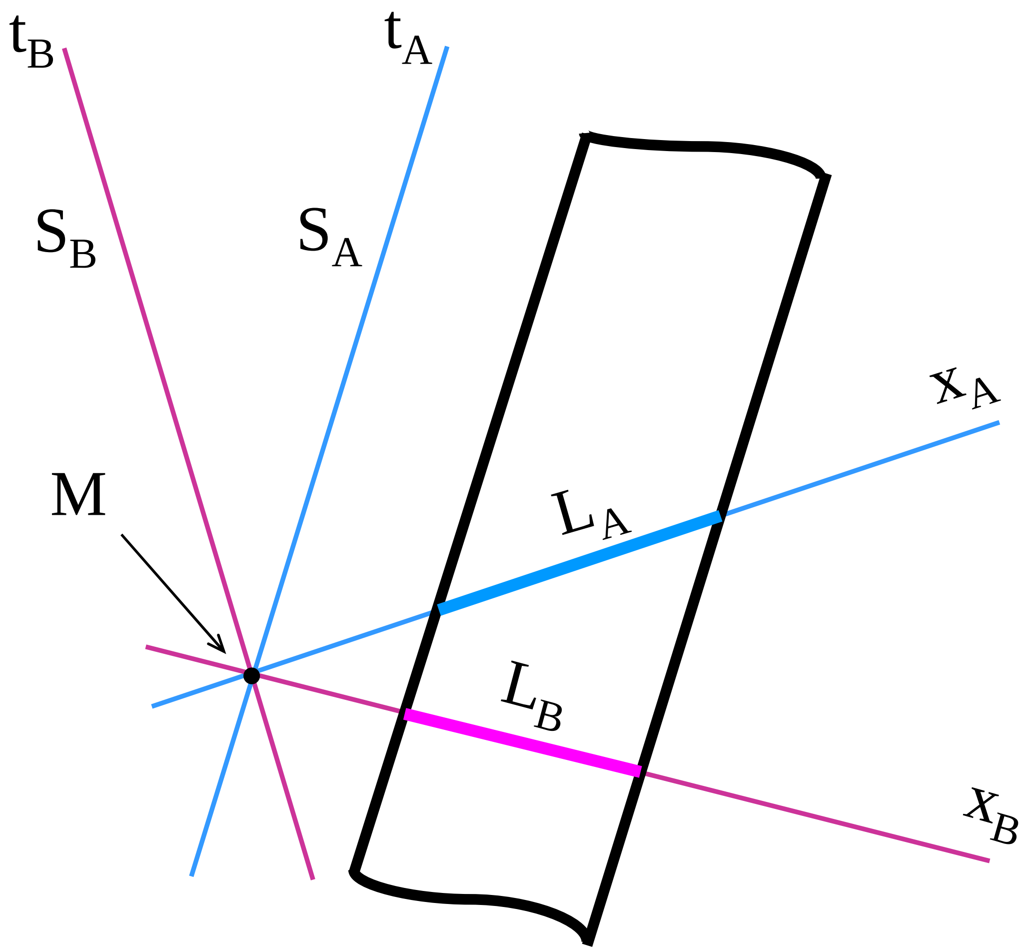
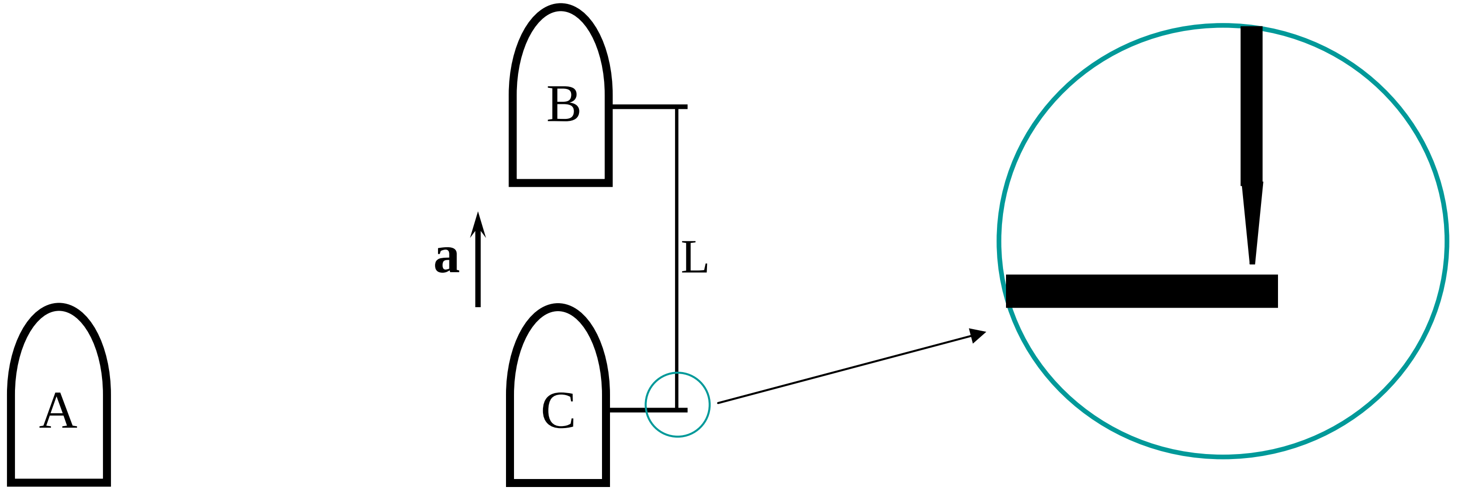


5. Bell implicitly assumed that all observers in A, B and C measure the ***same*** three-dimensional thread. But since a spatially extended three-dimensional object is defined in terms of ***simultaneity*** – all parts of the thread taken simultaneously at a given moment of time – it follows that while measuring the ***same*** thread two observers in relative motion, measure ***two different three-dimensional*** threads.

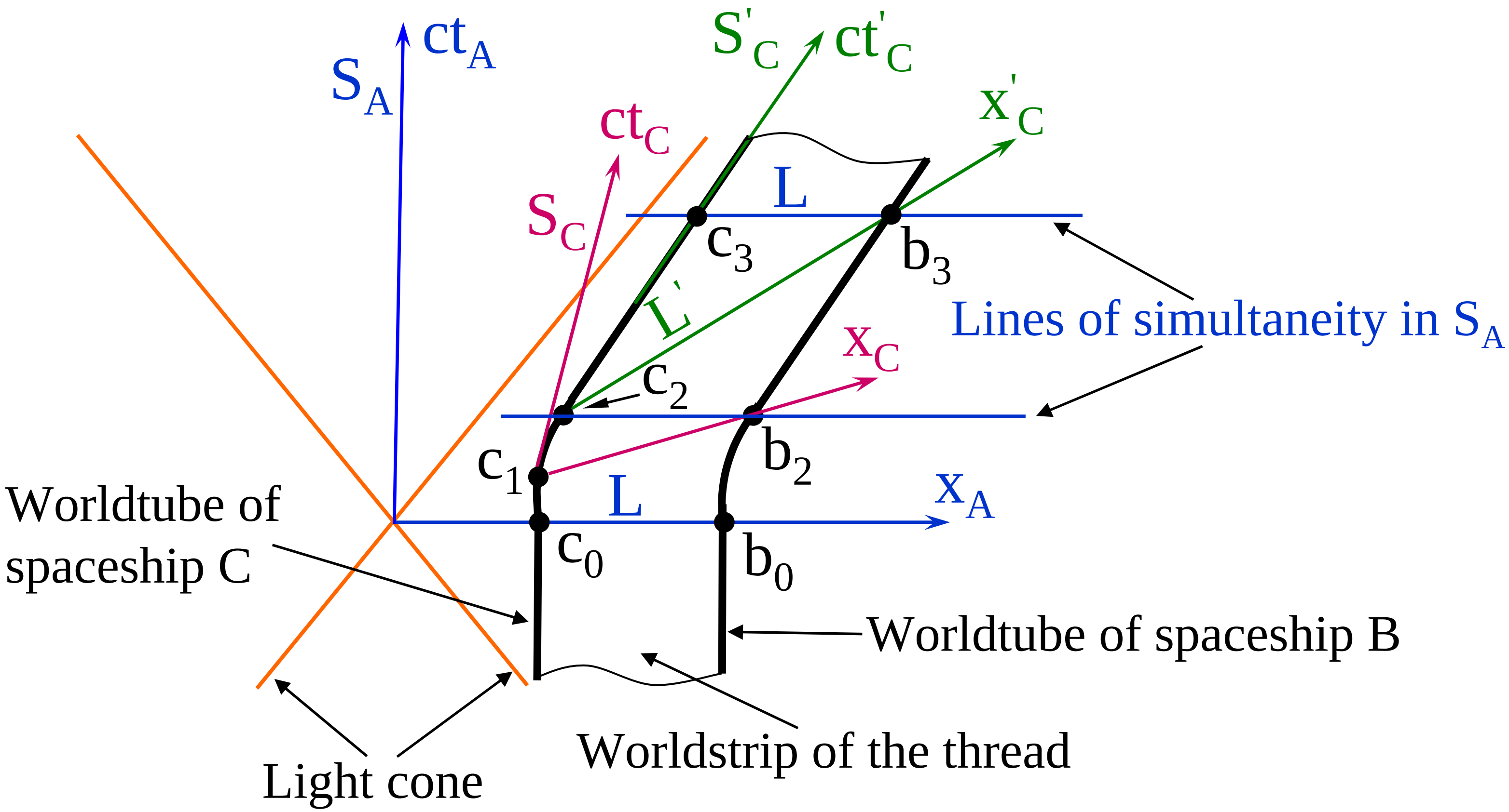
The measured lengths L_A and L_B do not represent real three-dimensional objects.

The only correct relativistic explanation of the constant distance L between B and C, as reckoned by an observer in A, is that the proper distance between B and C, measured by one of them, increases.

And it must increase for B and C exactly as much as it decreases for A due to length contraction in order that the distance L stays constant for an observer in A.



Instead of a thread consider a wire whose diameter is much smaller at its end that connects it to spaceship C. So when the wire breaks it breaks there.

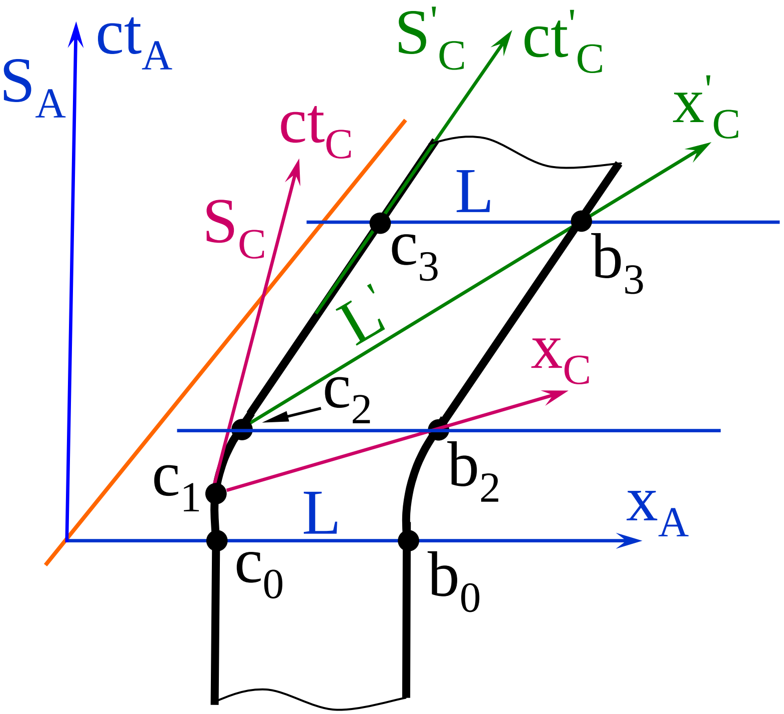


Spaceships B and C are represented by their worldtubes. The wire connecting B and C is represented by its worldstrip. The instantaneously comoving inertial reference frames S_C and S'_C corresponding to the events c_1 and c_2, respectively, are used to determine the increasing proper distance between B and C.

$$x'_{b_3} = \gamma(x_{b_3} - vt_{b_3})$$

$$x'_{c_2} = \gamma(x_{c_2} - vt_{c_2})$$

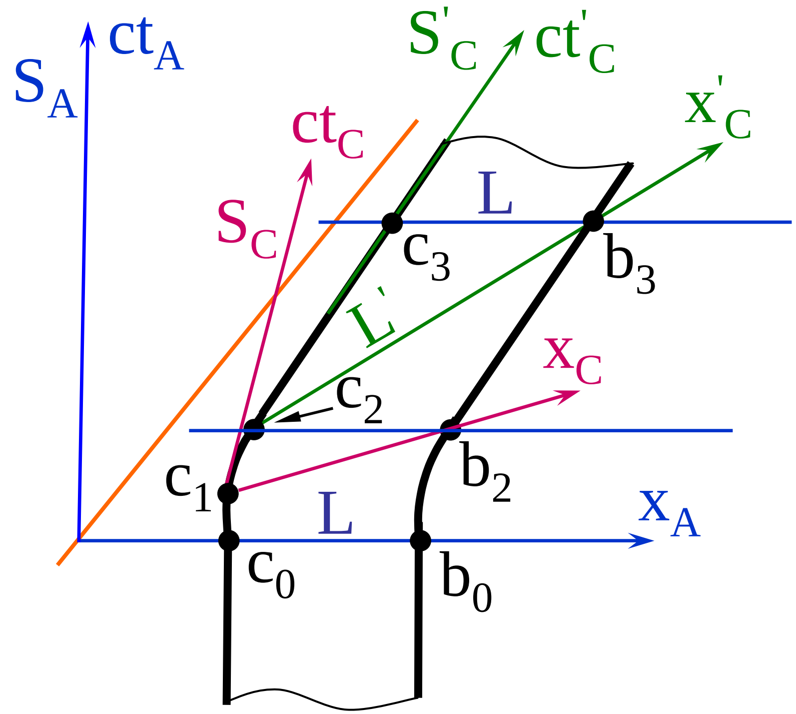
where $\gamma = (1 - v^2/c^2)^{-1/2}$



$$L' = x'_{b_3} - x'_{c_2}$$

$$L' = \gamma(x_{b_3} - x_{c_2}) - v\gamma(t_{b_3} - t_{c_2})$$

In S_A: $x_{b_3} = x_{c_3} + L$



$$x_{c_3} = x_{c_2} + v(t_{b_3} - t_{b_2})$$

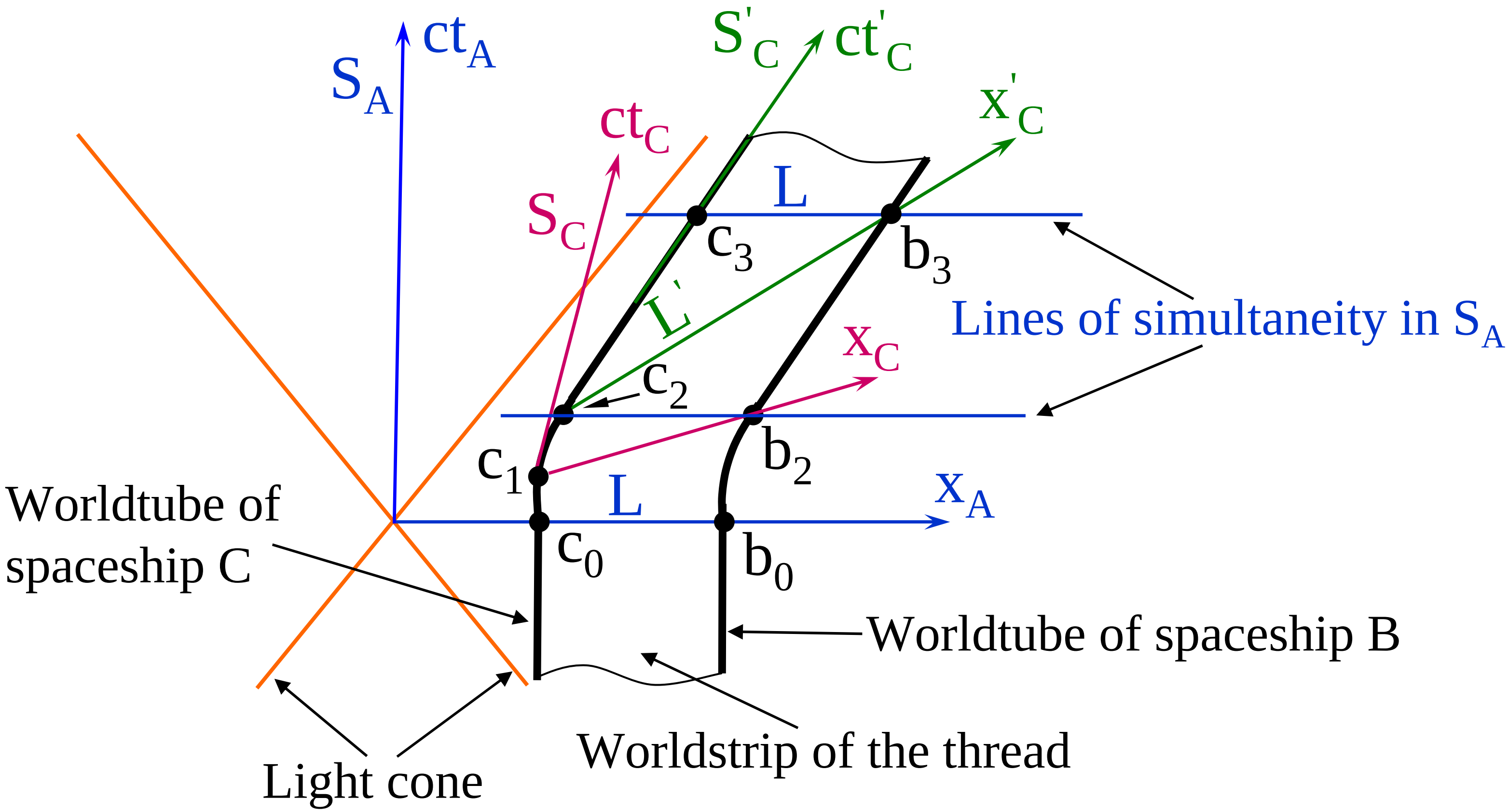
$$x_{b_3} = x_{c_2} + L + v(t_{b_3} - t_{b_2})$$

$$x_{b_3} - x_{c_2} = L + v(t_{b_3} - t_{c_2})$$

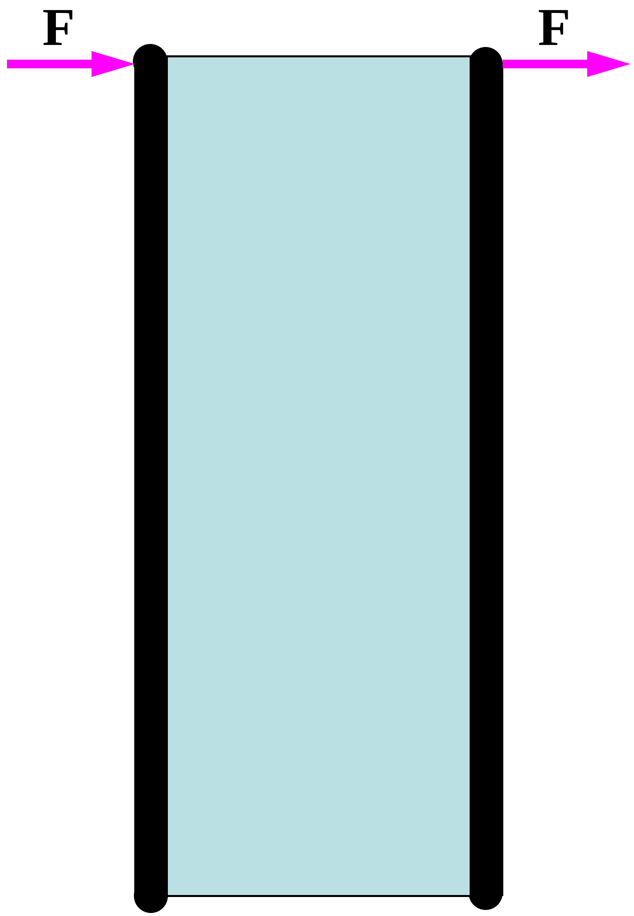
$$\boxed{t_{b_2} = t_{c_2} \text{ in } S_A}$$

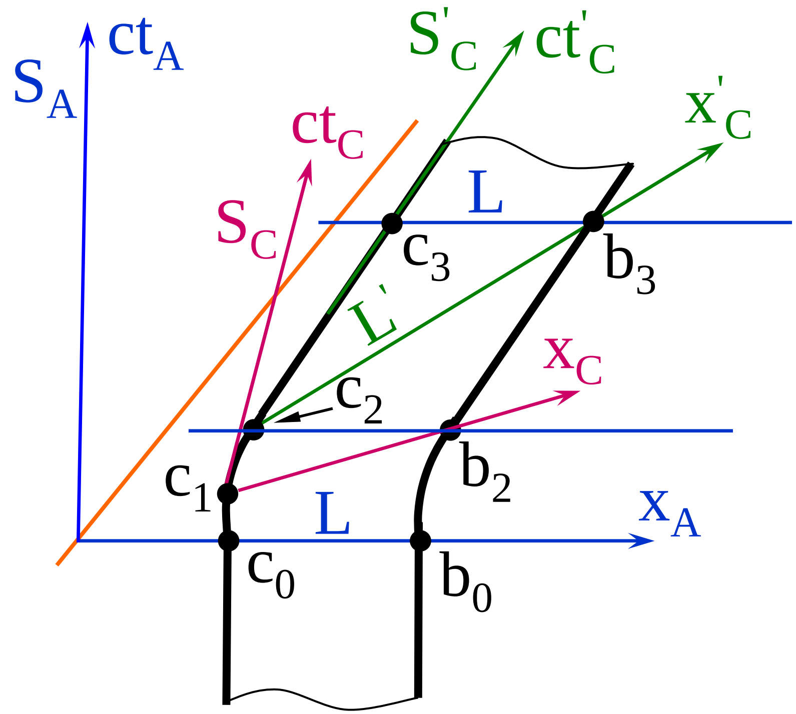
$$L' = \gamma(x_{b_3} - x_{c_2}) - v\gamma(t_{b_3} - t_{c_2}) \longrightarrow \boxed{L' = \gamma L}$$

$$\boxed{L_{contr} = \gamma^{-1} L' = L}$$



When the wire breaks at event c_2 the observers in A and C (or B) will measure ***two different three-dimensional wires*** – A will measure the 3D wire c_2b_2 (of length L), whereas C will measure the 3D wire c_2b_3 (of length L'). This means that ***B must exist at two events*** – at b_2 (for A) and at b_3 (for C). This shows that B's worldtube is a real 4D object.





If the proper distance between B and C depends on our choice, how could the increase of something that does not reflect anything objective cause the physical break of the wire?

Conclusion

- The very fact that after 50 years this paradox still causes confusion raises some questions about how relativity, and physics in general, should be taught. It mostly comes to the role of conceptual analyses in physics. Such analyses are regarded by many physicists as old-fashioned and even belonging to philosophy. The history of the fundamental breakthroughs in physics, however, convincingly demonstrates that conceptual analyses are physics at its best.

- The thread breaks due the increase of the proper distance between B and C, which is caused by the ***acceleration*** of B and C. So length contraction has nothing to do with the resolution of the paradox. Also, no stress is involved in length contraction.

- The real explanation of the paradox appears to be linked to the dimensionality of the objects involved in this paradox.

References

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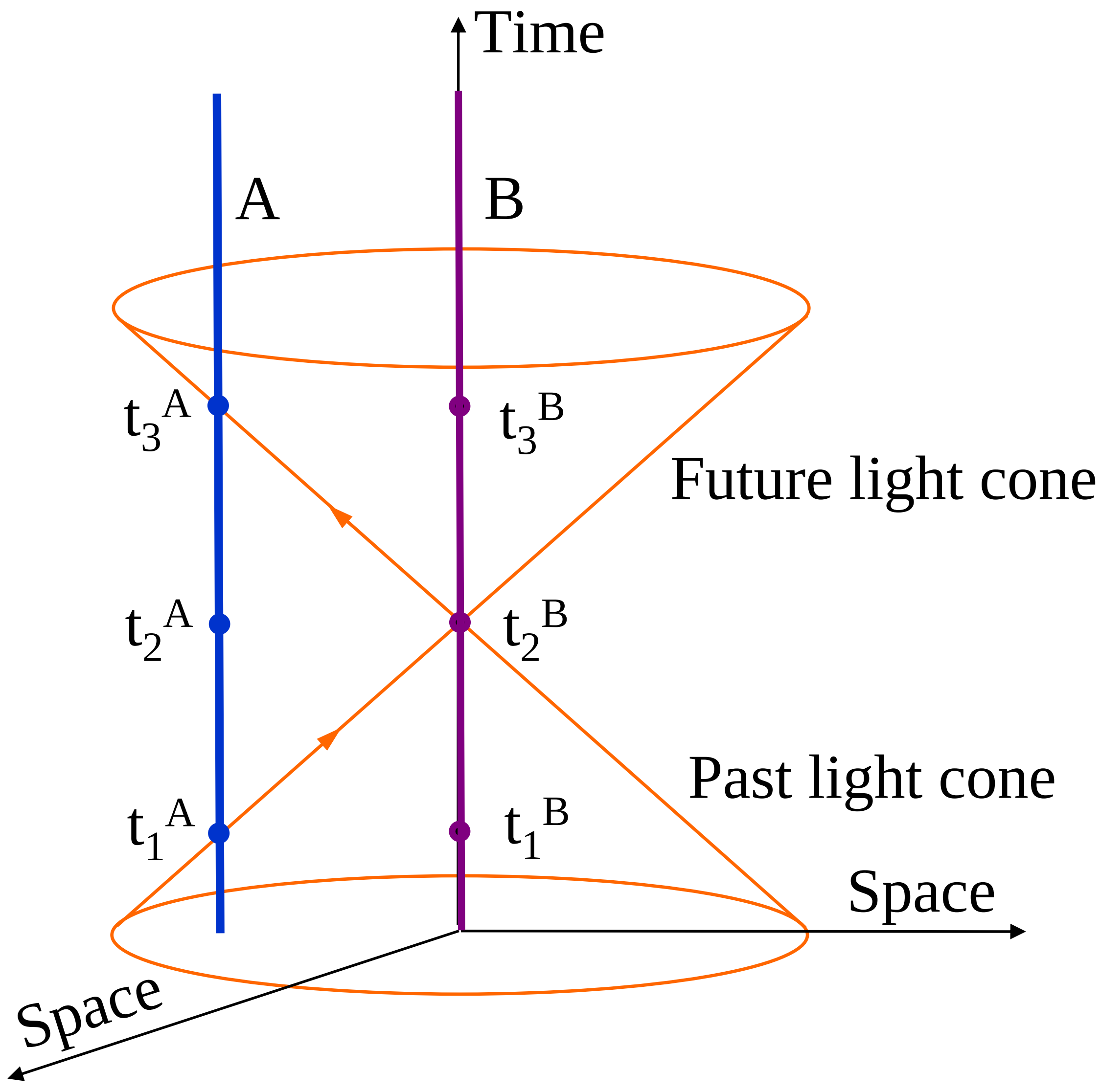
C.W. Misner, K.S. Thorne, J.A. Wheeler: *Gravitation* (Freeman, San Francisco 1973)

Vicious circle -- to determine whether two events are simultaneous we need to know the one-way velocity of light between them, but to determine the one-way velocity of light we need to know that the two events are simultaneous.



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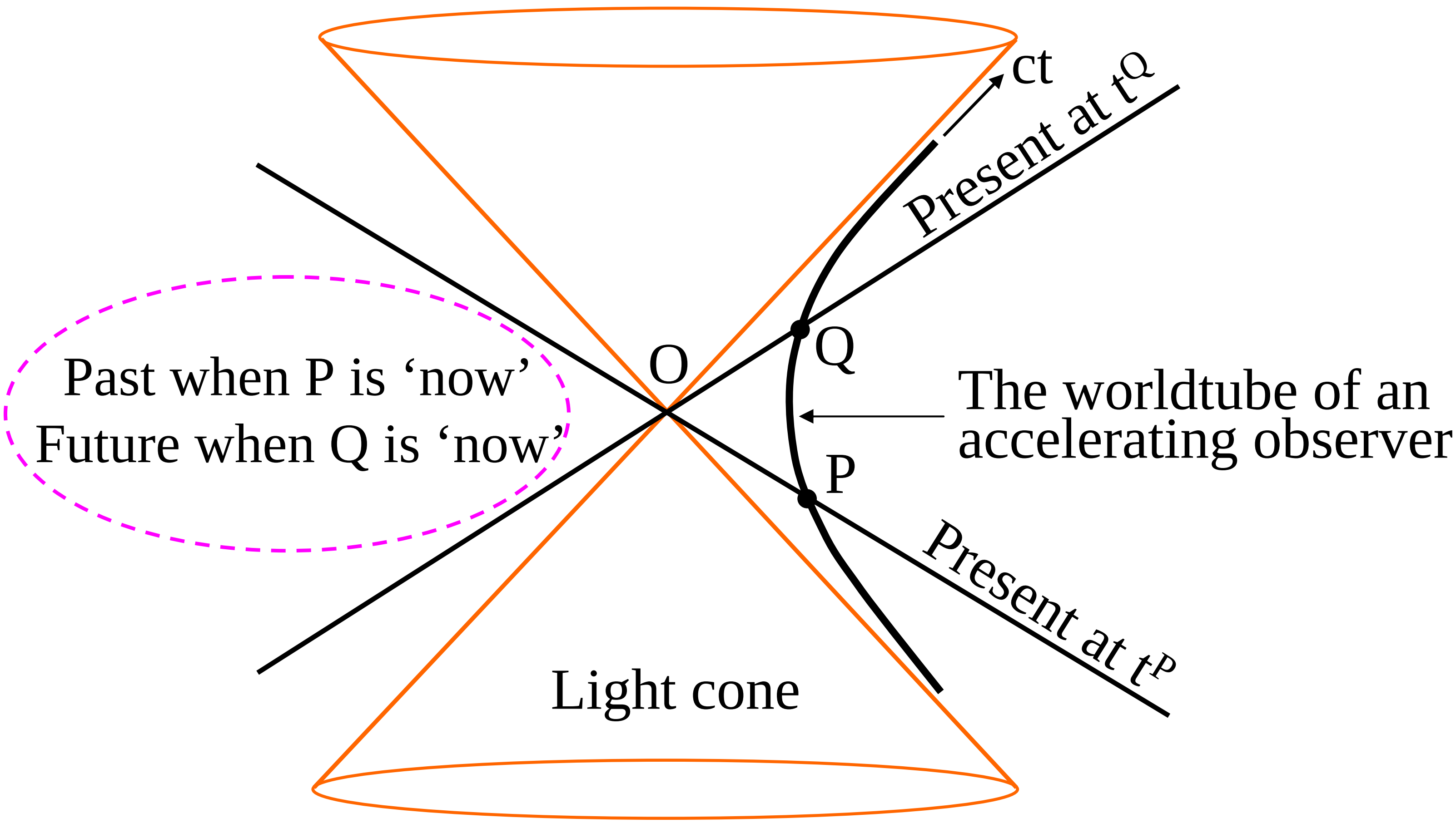
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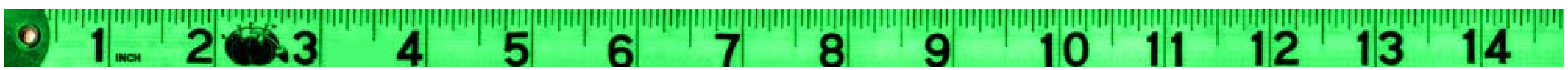
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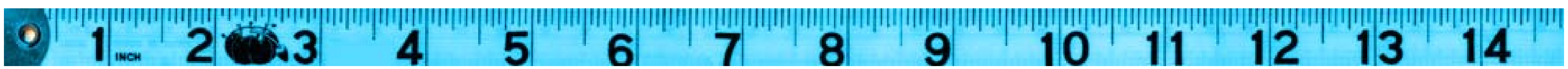
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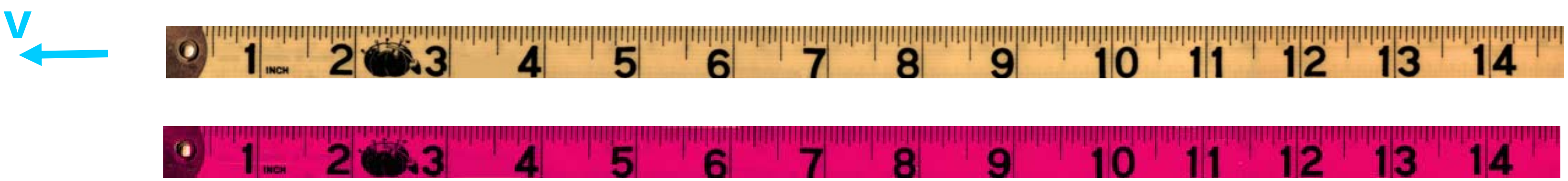
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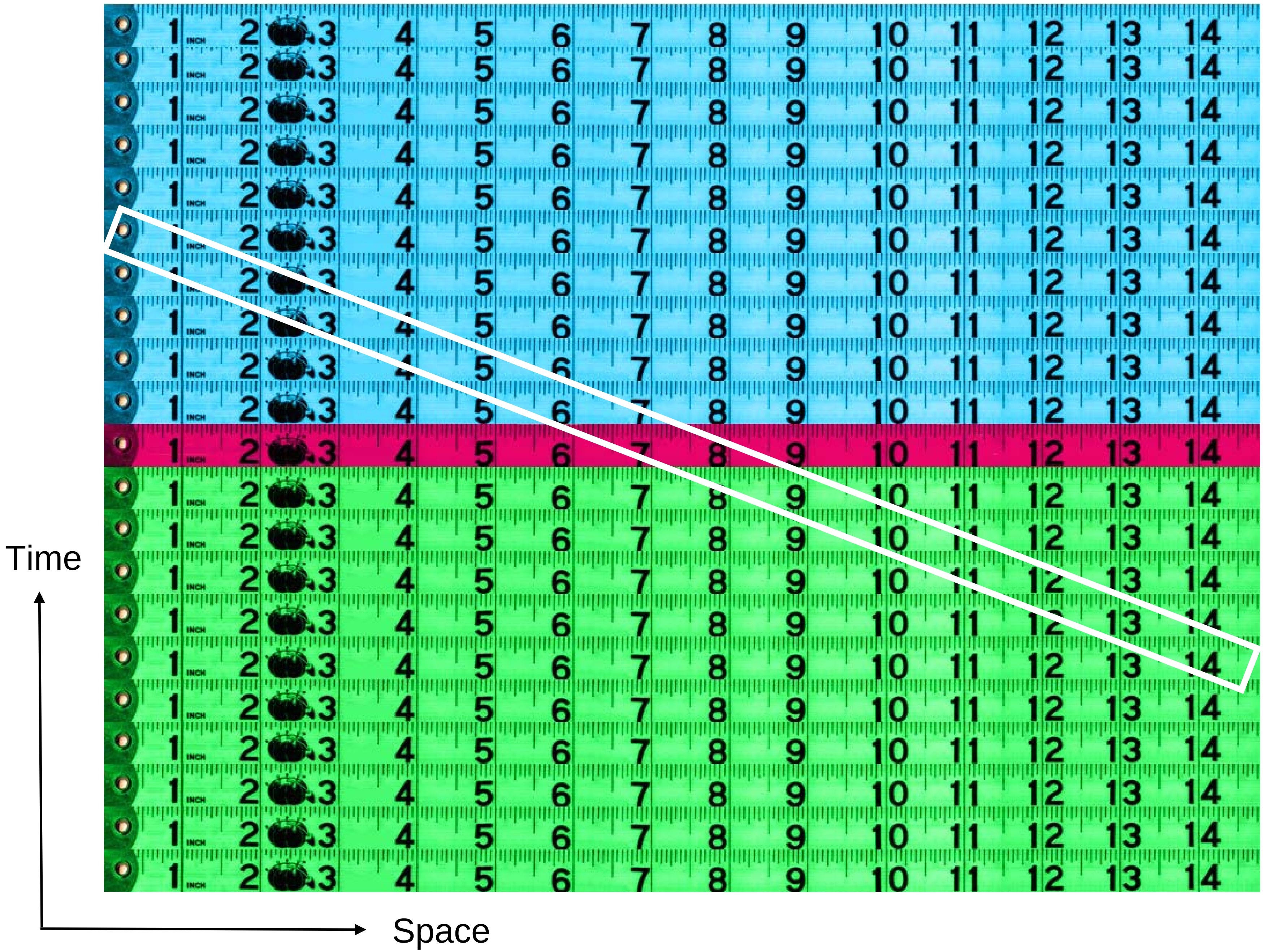
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